

2018 PiMC First Round

ALPHASTAR ACADEMY

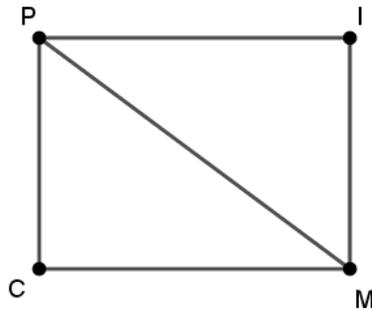
INDIVIDUAL TEST

INSTRUCTIONS

1. DO NOT OPEN THIS BOOKLET UNTIL YOUR PROCTOR TELLS YOU.
2. This is a 25 question test. Each question has a *single digit* answer: 0, 1, 2, 3, 4, 5, 6, 7, 8, or 9.
3. Mark your answer to each question on the Answer Form with a #2 pencil. Check the blackened circles for accuracy and erase errors and stray marks completely. Only answers properly marked on the answer form will be graded.
4. SCORING: You will receive 5 points for each correct answer, 1 point for each problem left unanswered, and 0 point for each incorrect answer.
5. As computational aids Rulers and Compasses are allowed. No calculators are allowed. No problem on the test will *require* the use of a calculator.
6. Figures are not necessarily drawn to scale.
7. When your proctor gives the signal, begin working on the problems. You will have **40 minutes** to complete the test.
8. After the exam, return your **Answer Form and Test Booklet** to your proctor.
9. You should NOT discuss any aspect of the exam questions with anyone until **January 29th**. Problems and solutions to the test will be posted on the contest web site after January 29th.

1. Evaluate $7 - 3 \times 2$.

2. In the figure below, what is the ratio of the area of the rectangle $PIMC$ to the area of the triangle PMC ?



3. How many positive integers less than 100 are divisible by 13?

4.

$$\frac{1}{3} + \frac{1}{1 + \frac{1}{2}} = ?$$

5. How many prime numbers are less than 20?

6. Find the number of 6 letter sequences of A 's and B 's such that no two consecutive letters are the same.

7. A positive integer is called a *perfect number* if the sum of its divisors, including the number itself, is equal to twice the number. For example, 28 is a perfect number because the sum of its divisors is $1 + 2 + 4 + 7 + 14 + 28 = 56 = 2 \times 28$. What is the smallest perfect number?

8. The ratio of the areas of two squares is 2 to 3. If the area of the smaller square is 6, what is the side length of the larger square?

9. Alice is doing quick maths. She is able to do 13 calculations per second. Her friend, Bob, joins her, doing 778 calculations per minute. After two minutes, how many more calculations has Alice done compared to the number of calculations Bob has done?

10. A worker earns \$1 on his first day, and on each of the following days, his pay is double the amount he earned the previous day. After several days, the worker saw that the total amount of money he earned was \$63. How many days did he work?

11. π is approximately 3.1415926535. After the decimal point, the first how many digits of π and $\frac{22}{7}$ are equal?

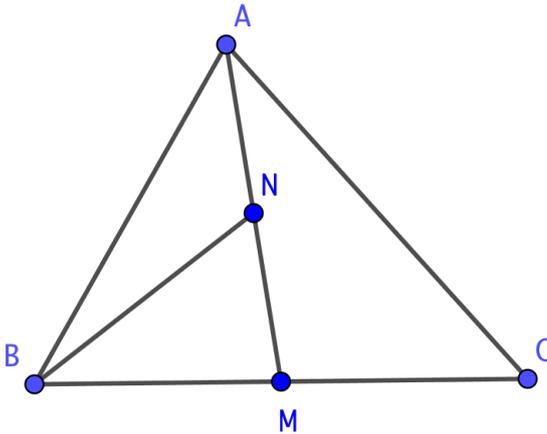
12. How many positive integers less than 40 are divisible by 2 and 5 but not 3?

13. Cindy takes five English tests during the school year, each scored out of 10 points. So far, Cindy has taken four tests and scored 5, 7, 9, and 10. If Cindy wants to have an average score of at least 8 on her five tests, what is the lowest she needs to score on her fifth test?

14. Tristan is taking a Math test with Algebra and Geometry sections. Tristan answered 48 of 75 Algebra questions correct and none of the Geometry questions correct. If he received a 60% on the Math test, how many Geometry questions are there on the test?

15. Find the units (ones) digit of $3^7 + 7^3$.

16. Let M be the midpoint of side BC on triangle ABC . Let N be the midpoint of AM . What is the ratio of the area of $\triangle ABC$ to the area of $\triangle BNM$?

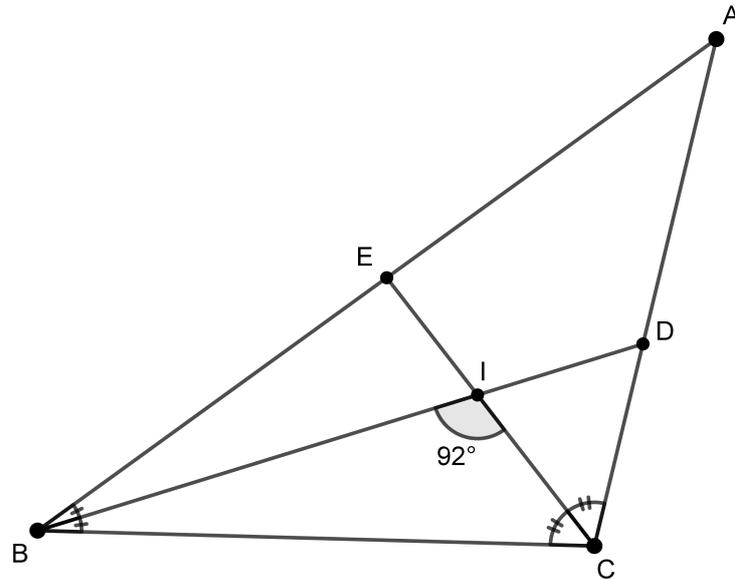


17. Jonathan the penguin is jumping along the number line. He starts at zero. Every time, he will jump either one unit to the right or one unit to the left. After seven jumps, at how many numbers could he possibly end up at?

18. What is the remainder when 201820182018 is divided by 9?

19. In how many ways can 6 be written as a sum of three positive integers where the order of the integers does not matter? For example, $1 + 1 + 4$ and $1 + 4 + 1$ are considered the same.
20. The start and end of a trail are 1 mile apart. Tom and Jerry both start at the beginning of the trail. Tom walks at a constant speed of 2 miles per hour and Jerry walks at a constant speed of 6 miles per hour. When Jerry reaches the end of the trail, he turns around and walks back until he reaches Tom; then he turns back and walks toward the end of the trail again. He repeats this until Tom has reached the end of the trail. How many miles does Jerry walk in total?
21. Victor wants to split an 8×12 rectangle into identical squares with integer side lengths. What is the smallest number of squares that he can get?
22. Edwin knows that the password to his computer is one of 1,000 common passwords. His password is equally likely to be any of these passwords. So he decides to guess different passwords randomly at a constant rate. If the probability that Edwin can guess his password in 10 minutes is $\frac{1}{20}$, he is guessing at a rate of how many passwords per minute?

23. In the figure below, BD and CE are angle bisectors and they intersect at I . If $\angle BIC = 92^\circ$, what is the measure of $\angle BAC$, in degrees?



24. An elementary school teacher chooses 4 students – Pam, Ian, Max, and Cai – to compete in a math contest. She notices that each student has at least one friend on the team and also all students have the same number of friends on the team. In how many ways is this possible? Note that friendship is always mutual. For example, if Pam is a friend of Ian, then Ian is a friend of Pam.
25. A group of boys build 5 squares, each with side length equal to the number of boys. A group of girls build 3 squares, each with side length equal to the number of girls. The total area of all 8 squares is 120. Find the total number of boys and girls.