2017 PiMC

FIRST ROUND

SOLUTIONS

1. (Ali Gurel) What is $2 + 2 \times 2$?

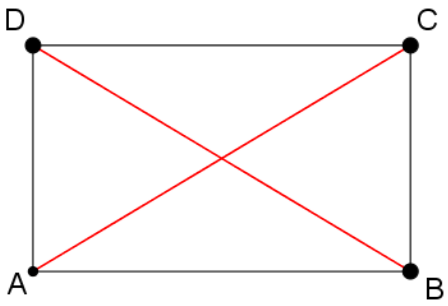
   Answer (6): $2 + 2 \times 2 = 2 + 4 = 6$.

2. (Handong Wang) Find the remainder when 2017 is divided by 67.

   Answer (7): When we do the long division, we find that $2017 = 67 \times 30 + 7$. So, the remainder is $\boxed{7}$.

3. (Freya Edholm) A *diagonal* of a polygon is a line segment between two vertices of the polygon that is NOT a side of the polygon. How many diagonals does a rectangle have?

   Answer (2): There are $\boxed{2}$ diagonals in a rectangle, as shown in red below: AC and BD.



4. (Andrew Lin) What is $(1 \times 2) + (2 \times 3) + (3 \times 4) - (4 \times 3) - (3 \times 2) - (2 \times 1)$?

   Answer (0): This is $2 + 6 + 12 - 12 - 6 - 2 = \boxed{0}$.

   Alternatively, note that when we multiply two numbers, it doesn't matter what order we multiply them in. Therefore, $1 \times 2 = 2 \times 1$, so those terms cancel. Similarly, the rest of the terms cancel, and the answer is $\boxed{0}$.

5. (Kaan Dokmeci) The *absolute value* of a number n, denoted $|n|$, is the distance between n and 0 on the number line. For example, $|11| = 11$ and $|-12| = 12$. Find $|2 \times (1 - 3)|$.

   Answer (4): $2 \times (1 - 3) = 2 \times (-2) = -4$ which is 4 away from 0. So the answer is $\boxed{4}$.

6. (Katherine Tian) Andy and Sandy are siblings. Andy is a boy and Sandy is a girl. Sandy has 1 brother and Andy has 3 sisters. How many siblings does Andy have?

   Answer (3): Sandy has only 1 brother: Andy. So, Andy is the only sibling who is a boy. Andy has 3 sisters (including Sandy) and no brothers. So the answer is $\boxed{3}$.

7. (Freya Edholm) Freya realizes that the letters in the word "WHO" can be rearranged to form the word "HOW". She notices that all *other* arrangements of those three letters result in bogus words. How many bogus words can she make by rearranging the letters W, H, O?

   Answer (4): There are 6 ways to arrange the letters W, H, and O:
   WHO
   WOH
   HOW
   HWO
   OHW
   OWH

   2 of the 6 arrangements, WHO and HOW, are not bogus words. So, Freya can make $\boxed{4}$ bogus words using the letters W, H, and O.

8. (Handong Wang) What is the hundreds digit of the first power of 2 greater than 600?

   Answer (0): When we list the powers of 2, we get: 2, 4, 8, 16, 32, 64, 128, 256, 512, 1024, etc. Then, we see that 1024 is the first power of 2 greater than 600. The hundreds digit of 1024 is $\boxed{0}$.

9. (Kaan Dokmeci, Freya Edholm) Edwin has a penguin machine. On day 1, the penguin machine produces 1 penguin. On day 2, it produces 2 penguins. On each day after the second day, the penguin machine produces one more penguin than it did on the previous day. How many days does it take the penguin machine to produce 36 penguins in total, including the first two days?

   Answer (8): After the first day, the penguin machine produced 1 penguin. After the second, it produced $1 + 2 = 3$ penguins total. We want to find a number n such that $1 + 2 + \ldots + n = 36$. We proceed using trial and error:
   $1 + 2 + 3 = 6$ (no)
   $1 + 2 + 3 + 4 = 10$ (no)
   $1 + 2 + 3 + 4 + 5 = 15$ (no)
   $1 + 2 + 3 + 4 + 5 + 6 = 21$ (no)
   $1 + 2 + 3 + 4 + 5 + 6 + 7 = 28$ (no)
   $1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 = 36$ (yes)
   So the answer is $\boxed{8}$.

10. (Ali Ersoz) There is a square with integer side lengths. The sum of its area and perimeter is 21. What is the side length of the square?

Answer (3): Since the side length is an integer and the area is side length squared, we only have to check up to side length 4 ($5^2 = 25 > 21$). For side length 1, we find that the sum of the perimeter and area is $1^2 + 4 \cdot 1 = 5$. For side length 2, this sum is $2^2 + 4 \cdot 2 = 12$. For side length 3, this sum is $3^2 + 4 \cdot 3 = 21$. Therefore, our answer is $\boxed{3}$.

11. (Tomas Choi) An *emirp* is a prime number that is also prime if you reverse the digits. For example, the prime number 107 is an emirp since 701 is also a prime number. What is the number of emirps less than 50?

    Answer (9): All emirps under 50 are: 2, 3, 5, 7, 11, 13, 17, 31, 37. There are 9 emirps less than 50. So the answer is $\boxed{9}$.

12. (Andrew Lin) What is the units digit of the number of ways that I can put 3 different envelopes into 7 different mailboxes if more than one envelope can be put into the same mailbox?

    Answer (3): I can put each envelope into 7 mailboxes. So, there are $7 \times 7 \times 7 = 343$ ways which has units digit $\boxed{3}$.

13. (Tomas Choi) Banana O'Reo can eat 240 sandwich cookies in 12 minutes. Apple O'Reo can eat the same number of sandwich cookies in 6 minutes. In how many minutes can Banana O'Reo and Apple O'Reo eat 240 sandwich cookies together?

    Answer (4): Banana O'Reo eats at a rate of $\frac{240}{12} = 20$ cookies per minute. Apple O'Reo eats at a rate of $\frac{240}{6} = 40$ cookies per minute. Working together, they eat at a rate of 60 cookies per minute. So, they can eat 240 cookies in $\frac{240}{60} = \boxed{4}$ minutes.

14. (Ali Ersoz) The circumference of a circle with radius 1 is equal to the perimeter of a square. Which integer is closest to the side length of the square?

    Answer (2): The circumference of the circle is $2\pi \times 1 = 2\pi$, which is equal to the perimeter of the square. Therefore, the square has side length $\frac{2\pi}{4} = \frac{\pi}{2}$, and the desired ratio is approximately $\frac{3.14}{2} = 1.57$, which is closest to $\boxed{2}$.

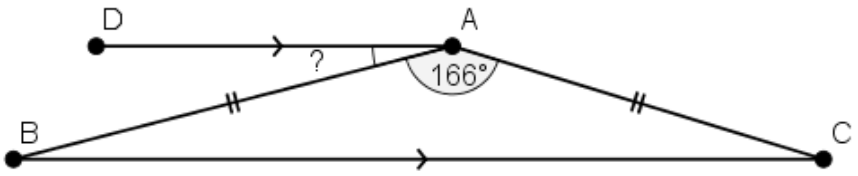
15. (Katherine Tian) Picasso and Lola ran four laps. Picasso, increasing his run time by 25% at each lap, ran the second lap in 80 seconds. Lola ran each lap in 90 seconds. If they started running at the same time, by how many seconds did Lola beat Picasso?

    Answer (9): Increasing by 25% is the same as multiplying by 1.25.

    The first lap took $\frac{80}{1.25} = 64$ seconds. The seconds lap took 80 seconds. The third lap took $80 \times 1.25 = 100$ seconds. The fourth lap took $100 \times 1.25 = 125$ seconds.

Picasso's total time is $64 + 80 + 100 + 125 = 369$ seconds. Lola's total time is $90 \times 4 = 360$ seconds. The difference is $\boxed{9}$ seconds.

16. (Kai-Siang Ang) In the figure below, $\triangle ABC$ is an isosceles triangle with $AB = AC$. DA is parallel to BC. If $\angle BAC = 166°$, then what is $\angle DAB$, in degrees?



Answer (7): $\angle DAB$ is equal to $\angle ABC$ and $\angle ACB$, which are equal, since ABC is isosceles. Call this angle measure x. Then, since the angles in $\triangle ABC$ sum to $180°$, $166 + 2x = 180$. So $2x = 14$ and $x = \boxed{7}$.

17. (Handong Wang) For the 25-question Individual Round, the Pi Math Contest has the following scoring system: 5 points for a correct answer, 1 point for a blank answer, and 0 points for an incorrect answer. Harry took the Individual Round and scored 50 points. At least how many questions did he answer correctly?

Answer (7): If Harry has no correct answers, then he can get at most 25 points. Note that he can get 25 points by leaving all answers blank. Changing each blank answer to a correct one would earn him 4 more points. Since he needs 25 more points to score 50, which is more than 6×4, he needs at least 7 correct answers. Finally, we note that 7 is possible since he can score 50 points with 7 correct, 15 blank, and 3 incorrect answers. So the answer is $\boxed{7}$.

18. (Stanley Wang) Chomas Toy and Wanli Stang are swimming laps. Chomas and Wanli start at opposite ends of the pool and start swimming towards each other at constant rates. The first time they meet, Chomas has swum two-fifths of a lap. Chomas stops swimming after 6 laps. If Wanli stops swimming at the same time, how many laps did he swim?

Answer (9): The first time they meet, if Chomas has swum $\frac{2}{5}$ of a lap, then Wanli has swum $\frac{3}{5}$ of a lap. Thus, Wanli's speed is $\frac{3}{2}$ times Chomas' speed. In the time it takes Chomas to swim 6 laps, Wanli will swim $6 \times \frac{3}{2} = \boxed{9}$ laps.

19. (Kevin Chang) Kevin is ordering pizza for the Pi Math Contest. He can order from 3 types of pizza: algebra pizza, geometry pizza, and number theory pizza. He needs to order at least 1 algebra pizza, at least 2 geometry pizzas, and at least 3 number theory pizzas. How many ways can Kevin order exactly 8 pizzas for the Pi Math Contest?

Answer (6): Because the types of 6 pizzas Kevin orders are already determined, the answer is equal to the number of ways Kevin can order exactly 2 pizzas for the Pi Math Contest without the earlier restrictions. He can either order 2 pizzas from one of the three types (3 ways: AA, GG, or NN) or order 1 pizza of one type and 1 of another (3 ways: AG, AN, or GN). There are $\boxed{6}$ total ways to do this.

20. (Ali Ersoz) The sum of two numbers is 10 and the sum of their reciprocals is 5. What is their product?

Answer (2): Let a and b be the two numbers. We know that $a + b = 10$ and $\frac{1}{a} + \frac{1}{b} = \frac{a+b}{ab} = 5$. It follows that $\frac{10}{ab} = 5$, so $ab = \boxed{2}$.

21. (Kai-Siang Ang) Alice and Bob are at the same house. Bob starts walking to school, and 10 minutes later, Alice starts walking to school. They both walk at constant speeds, but Alice walks 3 times as fast as Bob. They reach school at the same time. Alice was walking for how many minutes?

Answer (5): Since Alice walks 3 times as fast as Bob, Bob takes 3 times as long to walk the same distance as Alice. If Alice takes t minutes to walk to school, then Bob must take $3t$ minutes to walk to school. We are also given that Bob takes 10 more minutes to walk to school. Therefore, $3t = t + 10 \rightarrow 2t = 10 \rightarrow t = 5$. It is, Alice takes $\boxed{5}$ minutes to walk to school.

22. (Kai-Siang Ang) Find the difference between the two smallest numbers with exactly 6 positive factors.

Answer (6): 1 only has 1 factor. Primes only have 2 factors: 1 and themselves. We need to check 4, 6, 8, 9, 10, 12, 14, 15, 16, 18, etc. We see that the smallest two numbers with 6 divisors are 12 and 18. So the answer is $18 - 12 = \boxed{6}$.

23. (Katherine Tian) Anna wrote four letters to four different people and labeled four different envelopes with their names. However, Anna's sister, Hanna, wants to mix up the letters. In how many ways can Hanna put letters in envelopes so that no letter is in the envelope with the correct name?

Answer (9): Let the four letters be called A, B, C, and D. If the correct order is ABCD, we can list all the ways to arrange A, B, C, and D so that no letter ends up in its correct position:
BADC, BCDA, BDAC,
CADB, CDAB, CDBA,
DABC, DCAB, DCBA,
for a total of $\boxed{9}$.

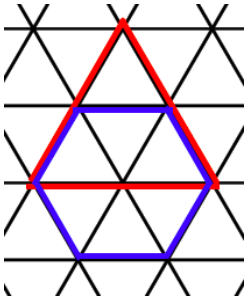
24. (Tomas Choi) Paco has 10 squares. He puts 1 grain of rice in the first square, 2 grains of rice in the second square, 4 grains of rice in the third square, and so on, doubling the number of grains of rice in the next square. Find the first (leftmost) digit of the the number of grains of rice he puts to fill all the squares.

Answer (1): For the first square, he needs 1 grain. For the first 2 squares, he needs 3 grains. For the first 3 squares, he needs 7 grains. We can see a pattern: each time he adds a square, he needs to double the number of grains and add 1. Thus, we continue doubling and adding 1 until we get that for 10 squares, we need 1023 grains of rice. So the answer is $\boxed{1}$.

Alternatively, note the pattern: $1 = 2^1 - 1$, $3 = 2^2 - 1$, $7 = 2^3 - 1$. So, we need $2^{10} - 1 = 1023$ grains of rice for 10 squares.

25. (Freya Edholm) An equilateral triangle and a regular hexagon have equal perimeters. If the area of the regular hexagon is 12, what is the area of the equilateral triangle?

Answer (8): Consider the following part of a triangular grid.



A hexagon is outlined in blue, and a triangle is outlined in red. We can see that they have the same perimeter (6 segments). Furthermore, the hexagon contains 6 small triangles with equal area. So each small triangles have area 2. The triangle contains 4 small triangles, so its area is $\boxed{8}$.