2017 PiMC

Final Round - Team Test

INSTRUCTIONS

1. DO NOT OPEN THIS BOOKLET UNTIL YOUR PROCTOR TELLS YOU.
2. This is a 15 question test. Team members may work together to solve the problems without disturbing other teams.
3. Write down your answer to each problem on the Team Answer Form. Only this form will be collected at the end of this round. All answers must be complete, legible and simplified to lowest terms.
4. SCORING: Your team will receive 20 points for each correct answer. No points will be given for problems left unanswered or incorrect answers.
5. No aids are permitted other than scratch paper, graph paper, rulers, compasses, protractors, and erasers. No calculators are allowed. No problems on the test will *require* the use of a calculator.
6. Figures are not necessarily drawn to scale.
7. When your proctor gives the signal, begin working on the problems. You will have **20 minutes** to complete the test.

1. Compute the sum $1+2-3+4+5-6+7+8-9+10+11-12$.

2. Alice and Bob love cats. Together they have 27 cats. Alice has 5 more cats than Bob. How many cats does Bob have?

3. What is the value of
$$\frac{3}{2} \times \frac{4}{3} \times \cdots \times \frac{99}{98} \times \frac{100}{99}?$$

4. My favorite number is a positive integer. When I square my favorite number, then divide by 4, and then subtract 7, I get 9. What is my favorite number?

5. If two apples and three oranges cost 13 cents and three apples and two oranges cost 12 cents, what is the cost of one apple and one orange?

6. What is the sum of the factors of 56?

7. The date January 17, 2017 has the property that when it is written in the standard mm/dd/yy format, 01/17/17, the product of the month and the day equals the last two digits of the year. How many dates in the years 2017, 2018, 2019 and 2020 have this property?

8. George picks a positive integer from 1 to 100. When George reverses the digits of his integer and subtracts it from his original number, he obtains a difference of 45. What is the largest number George could have picked?

9. What is

$$(2000 + 2001 + 2002 + \cdots + 2099) - (1900 + 1901 + 1902 + \cdots + 1999)?$$

10. How many positive integers less than 100 are not divisible by 4 or 7?

11. A rhombus has each side of length 5. If one diagonal has length 8, what is the area of the rhombus?

12. Spike walks at 2 miles per hour to get to work. As soon he gets there, however, he realizes that he forgot his laptop, so he runs back at 6 miles per hour, and then skips back to work at 3 miles per hour. What is his average speed?

13. Bob is ordering a pizza for himself. He has a choice of 3 crusts, 2 types of cheese, and 5 toppings. He needs to pick 1 crust, 1 type of cheese, and at least 1 topping. How many ways can he order the pizza?

14. On Planet Math, a "beauty" is a positive integer less than 1000 that is divisible by 2, 3, and 5, but not 4. How many *beauties* exist on Planet Math?

15. $\triangle ABC$ is a right triangle with a right angle at vertex B. Points P and Q are on sides AB and BC, respectively. Point M is the midpoint of PQ. If $PB = 8$ and $QC = 4$, what is the area of $\triangle PMC$?