

**2017 PiMC**

## FINAL ROUND - INDIVIDUAL TEST

## INSTRUCTIONS

1. DO NOT OPEN THIS BOOKLET UNTIL YOUR PROCTOR TELLS YOU.
2. This is a 25 question test. Each question has a *single digit* answer: 0, 1, 2, 3, 4, 5, 6, 7, 8, or 9.
3. Mark your answer to each question on the Answer Form with a #2 pencil. Check the blackened circles for accuracy and erase errors and stray marks completely. Only answers properly marked on the answer form will be graded.
4. SCORING: You will receive 5 points for each correct answer, 1 point for each problem left unanswered, and 0 point for each incorrect answer.
5. As computational aids Rulers and Compasses are allowed. No calculators are allowed. No problem on the test will *require* the use of a calculator.
6. Figures are not necessarily drawn to scale.
7. When your proctor gives the signal, begin working on the problems. You will have **40 minutes** to complete the test.
8. After the exam, return your **Answer Form** to your proctor. You can keep your test booklet.

1.  $5 - 3 \times 2 + 1 = ?$

2. Find the ones (units) digit of  $5^{18}$ .

$$5^{18} = \underbrace{5 \times 5 \times \cdots \times 5 \times 5}_{18 \text{ times}}.$$

3. At Poof School, 2 students disappear during each week of the school year. There were 43 students at Poof School at the beginning of the school year. After 17 full weeks, how many students have NOT yet disappeared from Poof School?

4. Kevin multiplies the first six positive integers. What is the smallest whole number that can be added to the product to get a multiple of 7?

5. What is the hundreds digit of  $1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 \times 8$ ?

6. Nathan runs four times as fast as he walks. If it takes Nathan 6 minutes to walk 1 mile, how many minutes does it take him to run 2 miles?

7. How many ways are there to choose 2 marbles from a bag of 4 different marbles, if the order of the selection is not important?

8. The area of a circle with radius 6 is how many times the area of a circle with radius 2?
  
  
  
  
  
  
  
  
  
  
9.  $ABCD$  is a rectangle with perimeter 12. There is a magic number such that side  $AB$  has length 3 less than the magic number and side  $BC$  has length 1 less than the magic number. Find the area of rectangle  $ABCD$ .
  
  
  
  
  
  
  
  
  
  
10. The number 4 has exactly three positive factors, namely 1, 2, and 4. What is the smallest positive integer that has exactly four positive factors?
  
  
  
  
  
  
  
  
  
  
11. Stanley selects a three digit positive integer and computes the sum of the digits. If Stanley obtains a sum of 25, how many three digit positive integers could Stanley have picked?
  
  
  
  
  
  
  
  
  
  
12. Let  $N$  be the smallest number greater than 1 that leaves a remainder of 1 when divided by 2, 3, 4, and 5. What is the sum of the digits of  $N$ ?
  
  
  
  
  
  
  
  
  
  
13. Brandon is writing a sequence of numbers whose first term is 1 and second term is 2. Each term after the second term is either the sum of the two previous terms if the sum is odd or the previous term if the sum is even. For example, if two consecutive terms in the sequence were 11 and 13, their sum would be even. Therefore, the next term would be 13. What is the 2017th term of Brandon's sequence?
  
  
  
  
  
  
  
  
  
  
14. How many ways are there to choose three people from a group of five people, namely Alex, Bob, Cindy, Diana, and Eric, to serve on a committee if Alex and Bob refuse to serve together?

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15. A  $4 \times 6$  rectangle is split into three squares each one having an integer side length. Find the sum of these three lengths.
16. If the area of a rectangle with integer side lengths is 36, how many distinct values are there for the perimeter of the rectangle?
17. Find the sum of all possible numbers that satisfy the following property: “36 divided by the number is equal to 24 minus three times the number”.
18. When you multiply three 7’s together, you get  $7 \times 7 \times 7 = 343$ , which leaves a remainder of 2 when divided by 11. What is the smallest number of 7’s you have to multiply together to get a remainder of 4 when divided by 11?
19. The square of 1 more than a positive number is equal to 17 more than twice the number. What is this number?
20. A year is *mathy* if the sum of its digits is 10. For example, 2017 is a *mathy* year because  $2 + 0 + 1 + 7 = 10$ . How many *mathy* years are there in this century?
21. Austin has 24 sticks, each 1 foot long. He uses them to build the largest possible cube. What is the volume of the cube, in cubic feet?

22. What is the units digit of

$$1^3 + 2^3 + 3^3 + \cdots + 19^3?$$

23. Harry wants to buy gum for an 11-day trip. He can buy gum in two different sizes, 9 sticks per pack or 14 sticks per pack. During the trip, Harry will chew the same number of sticks of gum every day. What is the smallest number of packs of gum he needs to buy if he does not want any leftovers at the end of the trip?

24. Pablo the Meerkat has 6 sticks of length 1, 2, 3, 4, 5, and 6. In how many ways can he select 3 of these sticks that form sides of a triangle with positive area, if the order of the selection of the 3 sticks is not important?

25. Ten overlapping  $1 \times 1$  squares are placed as shown below, where side lengths of the squares are trisected equally. What is the total area covered by these ten squares?

