PiMC 2016

Team Round

INSTRUCTIONS

1. DO NOT OPEN THIS BOOKLET UNTIL YOUR PROCTOR TELLS YOU.

2. This is a 15 question test. Team members may work together to solve the problems without disturbing other teams.

3. Write down your answer to each problem on the Team Answer Form. Only this form will be collected at the end of this round. All answers must be complete, legible and simplified to lowest terms.

4. SCORING: Your team will receive 20 points for each correct answer. No point will be given for problems left unanswered or incorrect answers.

5. No aids are permitted other than scratch paper, graph paper, rulers, compasses, protractors, and erasers. No calculators are allowed. No problems on the test will *require* the use of a calculator.

6. Figures are not necessarily drawn to scale.

7. When your proctor gives the signal, begin working on the problems. You will have **20 minutes** to complete the test.

8. You should NOT discuss any aspect of the exam problems with anyone until 5 pm, May 1, 2016.

1. Calculate
$$\frac{20 \times 22 \times 24}{10 \times 11 \times 12}.$$

2. Mark and John start a 40-mile race together. Mark runs at the constant rate of 10 miles per hour and John runs at the constant rate of 5 miles per hour. How many hours does Mark wait at the finish line before John reaches there?

3. Harry has red, blue, and green marbles. The number of red marbles is prime and the number of blue marbles is composite. The number of green marbles is neither prime nor composite. What is the smallest number of marbles that Harry could have?

4. The volume of a rectangular prism is the product of its length, width, and height. For example, if the length is 3, the width is 4, and the height is 5, then the volume of the prism is $3 \times 4 \times 5 = 60$. If you double the length, width, and height of a rectangular prism, by what percent does the volume increase?

5. Express $\dfrac{355}{113}$ as a decimal to the nearest hundred thousandth.

6. On the number line, what number, other than 1/5, is as far away from 1/6 as 1/5 is? Express your answer as a common fraction in simplest form.

7. A triangle is isosceles if two of its angles have the same measure. In an isosceles triangle $\triangle ABC$, $\angle A = 50°$. What is the sum of the largest and smallest possible degree measures of $\angle B$?

   Note: You may use the fact that the sum of the measures of three angles of a triangle is 180 degrees.

8. Allen has 5 identical potatoes. In how many ways can Allen put these potatoes in 3 different boxes if each box must contain at least one potato?

9. Four years ago, Julie was three times as old as Julia. Next year, Julia will be half as old as Julie. What is the sum of their ages now?

10. Christie is making paper cranes while Katrina unfolds them. It takes Christie 2 minutes to make one paper crane and it takes Katrina 15 seconds to unfold one. Christie starts with 21 cranes and makes more while Katrina unfolds them. How many seconds will pass before Christie has no cranes left?

11. Claire has a basket of Easter eggs. If she divides the eggs into groups of 4, she has 3 eggs left. If she divides the eggs into groups of 5, she has 4 eggs left. If she divides the eggs into groups of 6, she has 5 eggs left. What is the smallest possible number of eggs in her basket?

12. Call an integer "exquisite" if it leaves the same remainder when divided by 3 and 4. How many exquisite integers are there 1 through 100?

13. On a 4×4 chessboard, how many ways are there to place 2 identical rooks so that they do not attack one another? Rooks attack each other if they are placed in the same row or column.

14. How many two-digit numbers are divisible by 1 more than their tens digit and 1 more than their ones digit?

15. Tomas reads a math book. He reads 1 page on the first day, 2 pages on the second day, 3 pages on the third day and so on, each day reading one more page than the previous day. If the book has 2016 pages, how many days will it take Tomas to finish the book?