

PiMC 2016

INDIVIDUAL ROUND

INSTRUCTIONS

1. DO NOT OPEN THIS BOOKLET UNTIL YOUR PROCTOR TELLS YOU.
2. This is a 25 question test. Each question has a *single digit* answer: 0, 1, 2, 3, 4, 5, 6, 7, 8, or 9.
3. Mark your answer to each question on the Answer Form with a #2 pencil. Check the blackened circles for accuracy and erase errors and stray marks completely. Only answers properly marked on the answer form will be graded.
4. SCORING: You will receive 5 points for each correct answer, 1 point for each problem left unanswered, and 0 point for each incorrect answer.
5. No aids are permitted other than scratch paper, graph paper, rulers, compasses, protractors, and erasers. No calculators are allowed. No problems on the test will *require* the use of a calculator.
6. Figures are not necessarily drawn to scale.
7. When your proctor gives the signal, begin working on the problems. You will have **50 minutes** to complete the test.
8. You should NOT discuss any aspect of the exam problems with anyone until 5 pm, May 1, 2016.

1. What is 5% of 20?

2. $2 - 2 \times 2 + 2 = ?$

3. What is the tens digit of the product

$$3 \times 3 \times 3 \times 3 \times 3 \times 3?$$

4. Two different regular 6-sided dice are rolled. Among 36 possible outcomes how many of them have the two values shown on top of the dice sum to 10?

5. Simplify

$$8 \times \left(\frac{2}{6} + \frac{3}{24} + \frac{1}{24} \right).$$

6.

$$\star = \square + \square$$

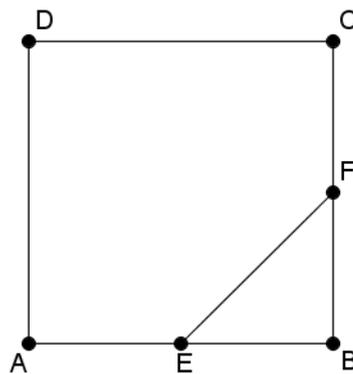
$$\square = \diamond \times \diamond$$

$$\diamond = \triangle \div \triangle$$

In the equations above, each symbol represents a positive integer. What is the value of \star ?

7. There are 25 prime numbers less than or equal to 100. What is the ones (units) digit of their product?
8. Danielle has two shirts, two skirts, and two hats. If an outfit consists of a shirt, a skirt, and a hat, how many different outfits can Danielle wear?
9. What is 20% of 25% of 40% of 50?
10. If x is a number such that $256^x = 4^{20}$, what is the value of x ?
11. Four more than double the number of dimes in a dollar is equal to the number of quarters in how many dollars?
12. Justin runs at the rate of 12 kilometers per hour. A mile is approximately 1.6 kilometers. At this rate, how many hours does it take Justin to run 30 miles? Round your answer to the nearest integer.

13. Danielle and Rachel have an equal number of berries. After Danielle eats two of her own berries, Celine comes and takes two-thirds of Danielle's remaining berries, and Rachel eats 80% of her own berries. At the end, Danielle and Rachel have the same number of berries again. How many berries did Danielle have at the beginning?
14. The area and the perimeter of a rectangle are numerically equal. If the length of the rectangle is 6, what is the width of this rectangle?
15. How many two-digit prime numbers have a ones (units) digit of 9?
16. Alice's rectangle has a perimeter of 14. Bob's rectangle is twice as long and a third as wide and has an area of 8. What is the longer side length of Alice's rectangle?
17. In the diagram below, $ABCD$ is a square with side length 4. E and F are midpoints of AB and BC . Find EF^2 .



18. In a game of Funkyball, a player throws a ball at the board. For every ball thrown, the player can either score 3 points or 5 points. What is the largest integer score that a player cannot attain? For example, a player can attain 13 points if he scores 3 points once and 5 points twice, but a player cannot possibly attain 1 point.

19. The area of a trapezoid is 20. The vertices of the trapezoid are at $(0,0)$, $(5,0)$, $(0,5)$, and $(x,5)$, where x is positive number. What is the value of x ?

20. A triangle has side lengths 2, 3, and 4. What is the ratio of the longest height to the shortest height?

21. An arithmetic sequence is a sequence of numbers where the difference between the consecutive terms is constant, such as 5, 7, 9, 11, \dots . The third term of an arithmetic sequence is 20 and the sum of the first eight terms is 244. What is the first term of the sequence?

22. The difference of two positive integers is 1. The difference of their squares is 77. What is the ones (units) digit of the larger number?

23. John flips a coin 6 times. What is the difference between the number of ways he can get 4 heads and the number of ways he can get 3 heads? For example, he can get 4 Heads with H H H H T T and 3 Heads with H H H T T T (H represents Heads and T represents Tails).

24. a, b, c, d are positive integers satisfying

$$(a \times b) + (b \times c) + (c \times d) + (d \times a) = 36.$$

How many possible values of $a + c$ are there?

25. In the equation below, each of the letters H, K, M, O, and T represents a different digit. H, M, and T cannot be zero. What is the value of M?

$$\begin{array}{rcccc} & M & O & T & H \\ + & T & O & O & K \\ \hline H & M & M & M & M \end{array}$$