2015 π Math Contest

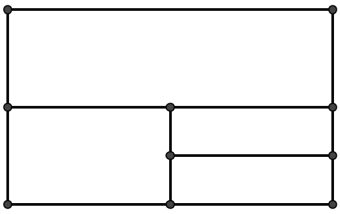
Individual Round

INSTRUCTIONS

1. DO NOT OPEN THIS BOOKLET UNTIL YOUR PROCTOR TELLS YOU.
2. This is a 25 question test. Each question has a *digit* answer: 0, 1, 2, 3, 4, 5, 6, 7, 8, or 9.
3. Mark your answer to each problem on the Answer Form with a #2 pencil. Check the blackened circles for accuracy and erase errors and stray marks completely. Only answers properly marked on the answer form will be graded.
4. SCORING: You will receive 5 points for each correct answer, 1 point for each problem left unanswered, and 0 point for each incorrect answer.
5. No aids are permitted other than scratch paper, graph paper, rulers, compasses, protractors, and erasers. No calculators are allowed. No problems on the test will *require* the use of a calculator.
6. Figures are not necessarily drawn to scale.
7. When your proctor gives the signal, begin working on the problems. You will have **50 minutes** to complete the test.

1. $7 - 3 \times 2 = ?$

2. In the diagram below, a rectangle is split into two halves by a horizontal line segment. Then the lower rectangle is split into two halves by a vertical line segment. Finally, the lower right rectangle is split into two halves by a horizontal line segment. What is the ratio of the area of the entire rectangle to the area of one of the two smallest rectangles?



3. A prime number is a whole number greater than 1 whose only two whole-number factors are 1 and itself. How many prime numbers are divisible by either 2, 3, or 4?

4. Kevin has a 65% off coupon which he wants to use to purchase new shoes. If the shoes originally cost \$20, how many dollars does he need to spend on the shoes using the coupon?

5. 1% of a number is what percent of 25% of that number?

6. The sum of Andrew's and Phoebe's ages is 16. Andrew was 2 years old when Phoebe was born. How old is Andrew now?

7. You can rearrange the letters in the "word" *bee* in three different ways: *bee*, *ebe*, and *eeb*. In how many ways can you rearrange the letters in the "word" *red*?

8. Waves crash on a beach at a rate of 1 wave every 35 seconds. If Aaron the squirrel sits on the beach for 5 minutes, how many waves will he see crashing onto the beach if Aaron sees his first wave at the end of the first 5 seconds?

9. In a group of 15 students, 5 like to play football, 6 like to play basketball and 7 like to play neither. How many students like to play both football and basketball?

10. Bob is mowing lawns over the summer. If he can mow a 10-foot by 10-foot lawn in 40 minutes, how many hours will it take him to mow a 12-foot by 25-foot lawn, given that he always mows at the same constant rate?

11. What is the sum of the digits of $1001 \times 1001 - 1000 \times 1000$?

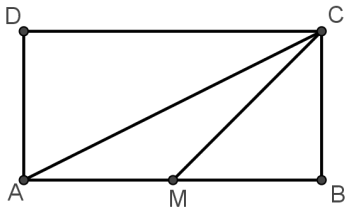
12. For how many minutes from midnight (12:00 AM) to noon (12:00 PM) on a standard 12-hour digital clock is the value of the hour equal to exactly twice the value of the minute? One example is the minute 4:02, since the hour (4) is twice the minute (2).

13. Adam is 13 years younger than Laura. Laura is twice as old as Jessica. Jessica is 5 years older than Kelvin. If Kelvin is 2 years old, how old is Adam?

14. Mario is collecting mushrooms. There are 3 kinds of mushrooms and he has to pay for them with coins. A red mushroom costs 5 coins, a blue mushroom costs 4 coins, and a yellow mushroom costs 1 coin. What is the smallest number of mushrooms he can collect with exactly 23 coins?

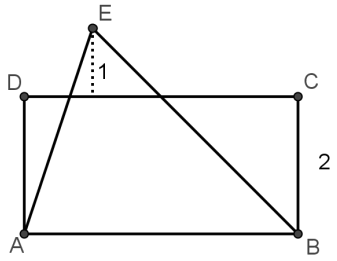
15. The sum of the factors of a positive integer is calculated by adding all the factors of the number. For example, the sum of the factors of 10 is $1 + 2 + 5 + 10 = 18$. For how many positive 1-digit numbers, $\{1, 2, 3, 4, 5, 6, 7, 8, 9\}$, is the sum of the factors one more than the digit itself?

16. In the diagram below, $ABCD$ is a rectangle and M is the midpoint of AB. If triangle AMC has area 2, what is the area of rectangle $ABCD$?



17. Allen flips a coin 9 times. If it lands heads, he gets 3 points. If it lands tails, he gets 1 point. After 9 flips, Allen has 13 points. How many times did the coin land tails?

18. In the image below, $ABCD$ is a rectangle with $BC = 2$ units. Point E is above segment CD such that the distance from E to CD is 1 unit. If the area of rectangle $ABCD$ is 8 square units, how many square units is the area of triangle ABE?



19. In how many ways can 9 be written as a sum of two or more prime numbers if the order of the primes in the sum does not matter? A prime number can be used more than once. For example, $9 = 2 + 2 + 5$ is one way to write it.

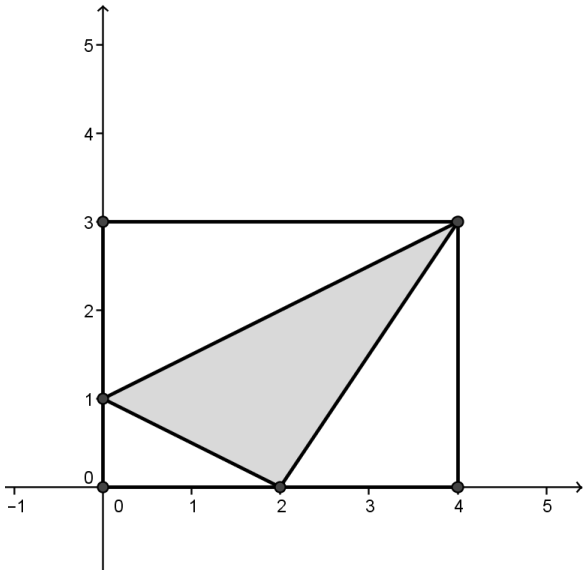
20. A squirrel family has less than 60 members. When the squirrels are arranged in 3 rows, 2 of them are left out. When they are arranged in 4 rows, 3 of them are left out. When they are arranged in 5 rows, none of them is left out. How many squirrels are in each row when they are arranged in 5 rows?

21. What is the ones digit of the sum of the first 30 positive integers?

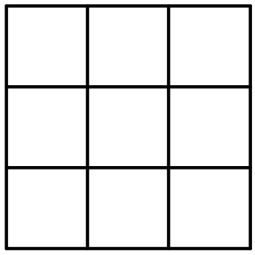
22. In heptagon $SWAPNIL$ with 7 sides, $\angle S = \angle W = \angle A = \angle P = \angle N = \angle I = 149$ degrees. How many degrees is $\angle L$?

    Note: You may use the fact that the sum of the three angles in a triangle is 180 degrees.

23. What is the area of the shaded triangle below?



24. What is the difference between the number of squares and the number of rectangles that are not squares in the diagram below?



25. Archibald and Cornelius each has a total of twelve pets, with each pet being either an octopus with eight legs or a turtle with four legs. Cornelius has twice as many octopi as Archibald does, and the total number of legs his pets have is twenty more than the total number of legs Archibald's pets have. How many octopi does Archibald have?