Topics & Sample Problems

MC35F (AMC 10/12 Advanced Fundamentals)

Part-II

MC35F-2 Geometry

Each chapter is followed by a short summary of topics covered in that chapter, along with one sample contest problem from that chapter. Supplemental topics introduced in MC35F are labeled with an asterisk (*).

Chapter 1: Angles

- Angle, triangle, polygon definitions, Alternate Interior Angle Theorem
- Inscribed Angle Theorem
- Defn. of cyclic quadrilateral, basic properties
- Miquel's Theorem (*)

(AMC12-2018-A23) In $\triangle PAT$, $\angle P = 36^\circ$, $\angle A = 56^\circ$, and $PA = 10$. Points U and G lie on sides $\overline{TP}$ and $\overline{TA}$, respectively, so that $PU = AG = 1$. Let M and N be the midpoints of segments $\overline{PA}$ and $\overline{UG}$, respectively. What is the degree measure of the acute angle formed by lines MN and PA?

Chapter 2: Pythagorean Theorem, Special Triangles

- Pythagorean Theorem
- Special right triangles (45-45-90, 30-60-90, and 15-75-90)
- Pythagorean triples, Euclid's Formula

(AMC10-2023-A17) Let $ABCD$ be a rectangle with $AB = 30$ and $BC = 28$. Point P and Q lie on $\overline{BC}$ and $\overline{CD}$ respectively so that all sides of $\triangle ABP$, $\triangle PCQ$, and $\triangle QDA$ have integer lengths. What is the perimeter of $\triangle APQ$?

Chapter 3: Similarity

- Similarity, congruence axioms (SSS, SAS, ASA, AA, HL)
- Angle Bisector Theorem

(AMC10-2016-A19) In rectangle $ABCD$, $AB = 6$ and $BC = 3$. Point E between B and C, and point F between E and C are such that $BE = EF = FC$. Segments $\overline{AE}$ and $\overline{AF}$ intersect $\overline{BD}$ at P and Q, respectively. The ratio $BP : PQ : QD$ can be written as $r : s : t$, where the greatest common factor of r, s and t is 1. What is $r + s + t$?

Chapter 4: Special Points

- Special points of a triangle (Centroid, incenter, circumcenter, orthocenter)

(AMC12-2011-A25) Triangle ABC has $\angle BAC = 60°$, $\angle CBA \leq 90°$, $BC = 1$, and $AC \geq AB$. Let H, I, and O be the orthocenter, incenter, and circumcenter of $\triangle ABC$, respectively. Assume that the area of pentagon $BCOIH$ is the maximum possible. What is $\angle CBA$?

Chapter 5: Length-1

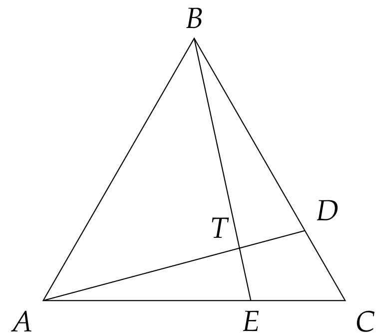
- Triangle inequality, Ravi substitution
- Stewart's, Apollonius' Theorems
- Power of a Point, Ptolemy's Theorem

(AMC10-2013-A23) In $\triangle ABC$, $AB = 86$, and $AC = 97$. A circle with center A and radius AB intersects $\overline{BC}$ at points B and X. Moreover $\overline{BX}$ and $\overline{CX}$ have integer lengths. What is BC?

Chapter 6: Length-2

- Mass points technique, levers, torque
- Ceva's and Menelaus' Theorems

(AMC10-2004-B20) In $\triangle ABC$ points D and E lie on BC and AC, respectively. If AD and BE intersect at T so that $\dfrac{AT}{DT} = 3$ and $\dfrac{BT}{ET} = 4$, what is $\dfrac{CD}{BD}$?



Chapter 7: Area-1

- Area formulas for polygons (triangles, rectangles, trapezoids, kites, etc.)
- Triangle area formulas (Heron's formula, $A = rs$, $A = \dfrac{abc}{4R}$)
- Viviani's Theorem (*)

(AMC12-2016-A15) Circles with centers P, Q and R, having radii 1, 2 and 3, respectively, lie on the same side of line l and are tangent to l at P', Q' and R', respectively, with Q' between P' and R'. The circle with center Q is externally tangent to each of the other two circles. What is the area of $\triangle PQR$?

Chapter 8: Area-2

- Area formulas for circles, sectors
- Ellipses, area of an ellipse

(AMC12-2006-B21) Rectangle $ABCD$ has area 2006. An ellipse with area 2006π passes through A and C and has foci at B and D. What is the perimeter of the rectangle? (The area of an ellipse is $ab\pi$ where $2a$ and $2b$ are the lengths of the axes.)

Chapter 9: Trigonometry-1

- Radians, right triangle and unit circle definitions of sin, cos, tan, etc.
- Basic trigonometric identities
- Heavier emphasis on geometry (compare with MC35F-1 Algebra, Chapter 10)

(AMC12-2024-B21) The measures of the smallest angles of three different right triangles sum to 90°. All three triangles have side lengths that are primitive Pythagorean triples. Two of them are 3-4-5 and 5-12-13. What is the perimeter of the third triangle?

Chapter 10: Trigonometry-2

- Extended Law of Sines
- Law of Cosines
- Proof of Stewart's Theorem (*)

(AMC12-2022-B19) In $\triangle ABC$ medians $\overline{AD}$ and $\overline{BE}$ intersect at G and $\triangle AGE$ is equilateral. Then $\cos(C)$ can be written as $\frac{m\sqrt{p}}{n}$, where m and n are relatively prime positive integers and p is a positive integer not divisible by the square of any prime. What is $m+n+p$?

Chapter 11: Analytic Geometry

- Basic definitions, theorems (slope, equation of a line, distance formula, etc.)
- Distance from a point to a line
- Shoelace Formula
- Conic sections (ellipses, parabolas, hyperbolas)

(AMC12-2015-A21) A circle of radius r passes through both foci of, and exactly four points on, the ellipse with equation $x^2 + 16y^2 = 16$. The set of all possible values of r is an interval $[a, b)$. What is $a + b$?

Chapter 12: 3D

- Polyhedra definitions, Euler's Polyhedral Formula
- Volume, surface area of various 3-dimensional solids (prisms, cones, pyramids, spheres, etc.)

(AMC10-2023-A18) A rhombic dodecahedron is a solid with 12 congruent rhombus faces. At every vertex, 3 or 4 edges meet, depending on the vertex. How many vertices have exactly 3 edges meet?

MC35F-2 Number Theory

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Chapter 1: Gauss Sums

- Sums of arithmetic sequences (e.g., sum of the first n positive integers)
- Summation (sigma) notation $\sum_{k=m}^{n} a_k$
- Sum of Squares, Sum of Cubes formulas

(AMC10-2023-B23) An arithmetic sequence of positive integers has $n \geq 3$ terms, initial term a, and common difference $d > 1$. Carl wrote down all the terms in this sequence correctly except for one term, which was off by 1. The sum of the terms he wrote was 222. What is $a + d + n$?

Chapter 2: Primes & Prime Factorization

- Defn. of divisor, factor, prime number, composite number
- Euclid's Lemma, Fundamental Theorem of Arithmetic, proof of infinitely many primes
- Legendre's Formula
- Dirichlet's Theorem (*)

(AIME-2006-II-3) Let P be the product of the first 100 positive odd integers. Find the largest integer k such that P is divisible by 3^k.

Chapter 3: Divisibility

- Divisibility rules for the integers 2 through 11, inclusive
- Divisibility test for larger integers, e.g., 101 (*)

(AMC10-2019-B14) The base-ten representation for 19! is $121{,}6T5{,}100{,}40M{,}832{,}H00$, where T, M, and H denote digits that are not given. What is $T + M + H$?

Chapter 4: Number & Sum of Divisors

- Definition of $d(n)$ and $\sigma(n)$, product (pi) notation $\prod_{k=m}^{n} a_k$
- Number and sum of divisors of a given positive integer n, given its prime factorization
- Product of the divisors of n
- Defn. of multiplicative function, multiplicativity of $d(n)$, $\sigma(n)$

(AMC10-2016-A22) For some positive integer n, the number $110n^3$ has 110 positive integer divisors, including 1 and the number $110n^3$. How many positive integer divisors does the number $81n^4$ have?

Chapter 5: Factoring Techniques

- Difference of Squares $(a^2 - b^2 = (a - b)(a + b))$
- Sum and Difference of Cubes $(a^3 \pm b^3)$
- Sophie-Germain Identity
- Simon's Favorite Factoring Trick

(AIME-2015-I-3) There is a prime number p such that $16p + 1$ is the cube of a positive integer. Find p.

Chapter 6: Number Bases

- Defn. of the base-b representation of a number n
- Converting integers between different number bases
- Decimal and fractional bases
- Fast base conversion (e.g., base 2 to base 16)

(AMC10-2013-A19) In base 10, the number 2013 ends in the digit 3. In base 9, on the other hand, the same number is written as $(2676)_9$ and ends in the digit 6. For how many positive integers b does the base-b-representation of 2013 end in the digit 3?

Chapter 7: GCD & LCM

- Defn. of greatest common divisor, least common multiple, relatively prime
- Properties of GCD and LCM
- Euclidean Algorithm

(AMC12-2020-A21) How many positive integers n are there such that n is a multiple of 5, and the least common multiple of 5! and n equals 5 times the greatest common divisor of 10! and n?

Chapter 8: Modular Arithmetic

- Defn. of congruence ($a \equiv b \pmod{m}$), basic properties of the $\equiv$ relation
- Finding remainders $\pmod{m}$ using patterns or properties of modular arithmetic
- Proof of the divisibility rules for 3 and 9
- Multiplicative inverse $a^{-1} \pmod{m}$
- Wilson's Theorem (*)

(AMC10-2022-A19) Define L_n as the least common multiple of all the integers from 1 to n inclusive. There is a unique integer h such that

$$\frac{1}{1}+\frac{1}{2}+\frac{1}{3}+\cdots+\frac{1}{17}=\frac{h}{L_{17}}.$$

What is the remainder when h is divided by 17?

Chapter 9: Fermat's Little Theorem

- Fermat's Little Theorem ($a^{p-1} \equiv 1 \pmod{p}$)
- Defn. of reduced residue system, proof of Fermat's Little Theorem

(AMC10-2017-B14) An integer N is selected at random in the range $1 \leq N \leq 2020$. What is the probability that the remainder when N^{16} is divided by 5 is 1?

Chapter 10: Euler's Totient Theorem

- Defn., properties of $\varphi(n)$
- Euler's Totient Theorem ($a^{\varphi(m)} \equiv 1 \pmod{m}$), connection to Fermat's Little Theorem

(AMC10-2024-B18) How many different remainders can result when the 100th power of an integer is divided by 125?

Chapter 11: Chinese Remainder Theorem

- Chinese remainder for a system of 2 modular congruences, and k modular congruences
- General formula for the solution to a system of 2 modular congruences
- Using the Chinese Remainder Theorem to compute an integer $\pmod{m}$, where m is composite

(AMC10-2017-B23) Let $N = 123456789101112\dots4344$ be the 79-digit number that is formed by writing the integers from 1 to 44 in order, one after the other. What is the remainder when N is divided by 45?

Chapter 12: Diophantine Equations

- Linear Diophantine equations ($ax + by = c$), Bézout's Identity
- Postage Stamp Theorem
- Sum of Squares Theorem
- Higher-degree Diophantine equations

(AMC12-2023-B16) In the state of Coinland, coins have values 6, 10, and 15 cents. Suppose x is the value in cents of the most expensive item in Coinland that cannot be purchased using these coins with exact change. What is the sum of the digits of x?