Topics & Sample Problems

MC30F (AMC 10/12 Basic Fundamentals)

alphastar academy

Part-II

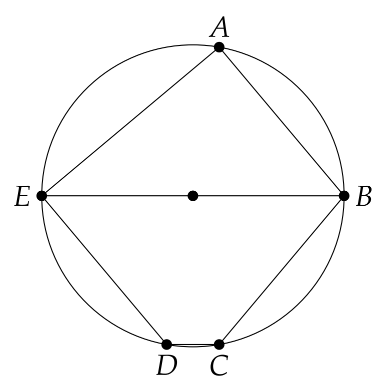
MC30F-2 Geometry

Each chapter is followed by a short summary of topics covered in that chapter, along with one sample contest problem from that chapter.

Chapter 1: Angles

- Angle, triangle, polygon definitions, Alternate Interior Angle Theorem
- Inscribed Angle Theorem
- Defn. of cyclic quadrilateral, basic properties

(AMC10-2011-B17) In the given circle, the diameter $\overline{EB}$ is parallel to $\overline{DC}$, and $\overline{AB}$ is parallel to $\overline{ED}$. The angles AEB and ABE are in the ratio $4:5$. What is the degree measure of angle BCD?



Chapter 2: Pythagorean Theorem, Special Triangles

- Pythagorean Theorem
- Special right triangles (45-45-90, 30-60-90, and 15-75-90)
- Pythagorean triples, Euclid's Formula

(AMC10-Spring-2021-B14) Three equally spaced parallel lines intersect a circle, creating three chords of lengths $38, 38$, and 34. What is the distance between two adjacent parallel lines?

Chapter 3: Similarity

- Similarity, congruence axioms (SSS, SAS, ASA, AA, HL)
- Angle Bisector Theorem

(AMC10-2010-A16) Nondegenerate $\triangle ABC$ has integer side lengths, $\overline{BD}$ is an angle bisector, $AD = 3$, and $DC = 8$. What is the smallest possible value of the perimeter?

Chapter 4: Special Points

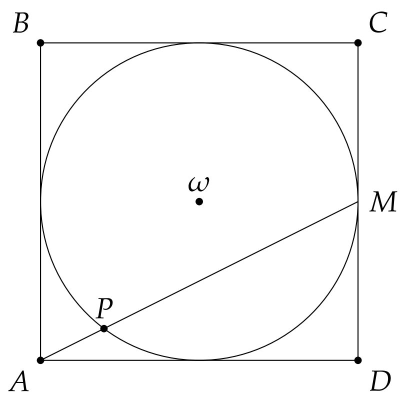
- Special points of a triangle (Centroid, incenter, circumcenter, orthocenter)

(AMC12-2011-A13) Triangle ABC has side-lengths $AB = 12$, $BC = 24$, and $AC = 18$. The line through the incenter of $\triangle ABC$ parallel to $\overline{BC}$ intersects $\overline{AB}$ at M and $\overline{AC}$ at N. What is the perimeter of $\triangle AMN$?

Chapter 5: Length-1

- Triangle inequality, Ravi substitution
- Stewart's, Apollonius' Theorems
- Power of a Point, Ptolemy's Theorem

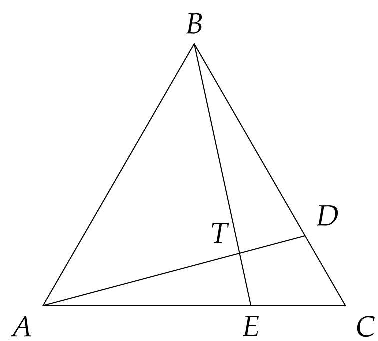
(AMC12-2020-B10) In unit square $ABCD$, the inscribed circle ω intersects $\overline{CD}$ at M, and $\overline{AM}$ intersects ω at a point P different from M. What is AP?



Chapter 6: Length-2

- Mass points technique, levers, torque
- Ceva's and Menelaus' Theorems

(AMC10-2004-B20) In $\triangle ABC$ points D and E lie on BC and AC, respectively. If AD and BE intersect at T so that $\dfrac{AT}{DT} = 3$ and $\dfrac{BT}{ET} = 4$, what is $\dfrac{CD}{BD}$?



Chapter 7: Area-1

- Area formulas for polygons (triangles, rectangles, trapezoids, kites, etc.)
- Triangle area formulas (Heron's formula, $A = rs$, $A = \dfrac{abc}{4R}$)

(AMC12-2020-A14) Regular octagon $ABCDEFGH$ has area n. Let m be the area of quadrilateral $ACEG$. What is $\dfrac{m}{n}$?

Chapter 8: Area-2

- Area formulas for circles, sectors
- Ellipses, area of an ellipse

(AMC12-2015-B14) A circle of radius 2 is centered at A. An equilateral triangle with side 4 has a vertex at A. What is the difference between the area of the region that lies inside the circle but outside the triangle and the area of the region that lies inside the triangle but outside the circle?

Chapter 9: Trigonometry-1

- Radians, right triangle and unit circle definitions of sin, cos, tan, etc.
- Basic trigonometric identities
- Heavier emphasis on geometry (compare with MC30F-1 Algebra, Chapter 10)

(AMC12-2012-A10) A triangle has area 30, one side of length 10, and the median to that side of length 9. Let θ be the acute angle formed by that side and the median. What is $\sin\theta$?

Chapter 10: Trigonometry-2

- Extended Law of Sines
- Law of Cosines

(AMC12-2003-B21) An object moves 8 cm in a straight line from A to B, turns at an angle α, measured in radians and chosen at random from the interval $(0, \pi)$, and moves 5 cm in a straight line to C. What is the probability that $AC < 7$?

Chapter 11: Analytic Geometry

- Basic definitions, theorems (slope, equation of a line, distance formula, etc.)
- Distance from a point to a line
- Shoelace Formula
- Conic sections (ellipses, parabolas, hyperbolas)

(AMC10-2013-A18) Let points $A = (0,0)$, $B = (1,2)$, $C = (3,3)$, and $D = (4,0)$. Quadrilateral $ABCD$ is cut into equal area pieces by a line passing through A. This line intersects $\overline{CD}$ at point $\left(\frac{p}{q}, \frac{r}{s}\right)$, where these fractions are in lowest terms. What is $p + q + r + s$?

Chapter 12: 3D

- Polyhedra definitions, Euler's Polyhedral Formula
- Volume, surface area of various 3-dimensional solids (prisms, cones, pyramids, spheres, etc.)

(AMC10-2012-B17) Jesse cuts a circular paper disk of radius 12 along two radii to form two sectors, the smaller having a central angle of 120 degrees. He makes two circular cones, using each sector to form the lateral surface of a cone. What is the ratio of the volume of the smaller cone to that of the larger?

MC30F-2 Number Theory

Each chapter is followed by a short summary of topics covered in that chapter, along with one sample contest problem from that chapter.

Chapter 1: Gauss Sums

- Sums of arithmetic sequences (e.g., sum of the first n positive integers)
- Summation (sigma) notation $\sum_{k=m}^{n} a_k$
- Sum of Squares, Sum of Cubes formulas

(AMC12-2002-B13) The sum of 18 consecutive positive integers is a perfect square. The smallest possible value of this sum is

Chapter 2: Primes & Prime Factorization

- Defn. of divisor, factor, prime number, composite number
- Euclid's Lemma, Fundamental Theorem of Arithmetic, proof of infinitely many primes
- Legendre's Formula

(AMC10-2018-B11) Which of the following expressions is never a prime number when p is a prime number?

Chapter 3: Divisibility

- Divisibility rules for the integers 2 through 11, inclusive

(AMC10-2021-Fall-A5) The six-digit number $\underline{2}\,\underline{0}\,\underline{2}\,\underline{1}\,\underline{0}\,\underline{A}$ is prime for only one digit A. What is A?

Chapter 4: Number & Sum of Divisors

- Definition of $d(n)$ and $\sigma(n)$, product (pi) notation $\prod_{k=m}^{n} a_k$
- Number and sum of divisors of a given positive integer n, given its prime factorization
- Product of the divisors of n
- Defn. of multiplicative function, multiplicativity of $d(n)$, $\sigma(n)$

(AMC10-2024-B8) Let N be the product of all the positive integer divisors of 42. What is the units digit of N?

Chapter 5: Factoring Techniques

- Difference of Squares $(a^2 - b^2 = (a-b)(a+b))$
- Sum and Difference of Cubes $(a^3 \pm b^3)$
- Sophie-Germain Identity
- Simon's Favorite Factoring Trick

(AMC10-2014-B17) What is the greatest power of 2 that is a factor of $10^{1002} - 4^{501}$?

Chapter 6: Number Bases

- Defn. of the base-b representation of a number n
- Converting integers between different number bases
- Decimal and fractional bases
- Fast base conversion (e.g., base 2 to base 16)

(AMC12-2005-A19) A faulty car odometer proceeds from digit 3 to digit 5, always skipping the digit 4, regardless of position. If the odometer now reads 002005, how many miles has the car actually traveled?

Chapter 7: GCD & LCM

- Defn. of greatest common divisor, least common multiple, relatively prime
- Properties of GCD and LCM
- Euclidean Algorithm

(AMC10-2017-A16) There are 10 horses, named Horse 1, Horse 2, $\ldots$, Horse 10. They get their names from how many minutes it takes them to run one lap around a circular race track: Horse k runs one lap in exactly k minutes. At time 0 all the horses are together at the starting point on the track. The horses start running in the same direction, and they keep running around the circular track at their constant speeds. The least time $S > 0$, in minutes, at which all 10 horses will again simultaneously be at the starting point is $S = 2520$. Let $T > 0$ be the least time, in minutes, such that at least 5 of the horses are again at the starting point. What is the sum of the digits of T?

Chapter 8: Modular Arithmetic

- Defn. of congruence ($a \equiv b \pmod{m}$), basic properties of the $\equiv$ relation
- Finding remainders $\pmod{m}$ using patterns or properties of modular arithmetic
- Proof of the divisibility rules for 3 and 9
- Multiplicative inverse $a^{-1} \pmod{m}$

(BMT-2015-Individual-9) The number 2^{29} has a 9-digit decimal representation that contains all but one of the 10 (decimal) digits. Determine which digit is missing.

Chapter 9: Fermat's Little Theorem

- Fermat's Little Theorem ($a^{p-1} \equiv 1 \pmod{p}$)
- Defn. of reduced residue system, proof of Fermat's Little Theorem

(Lehigh MC-2016-19) What is the remainder when $17^{17^{17}}$ is divided by 7?

Chapter 10: Euler's Totient Theorem

- Defn., properties of $\varphi(n)$
- Euler's Totient Theorem ($a^{\varphi(m)} \equiv 1 \pmod{m}$), connection to Fermat's Little Theorem

(SMT-2021-General-6) How many natural numbers less than 2021 are coprime to 2021?

Chapter 11: Chinese Remainder Theorem

- Chinese remainder for a system of 2 modular congruences, and k modular congruences
- General formula for the solution to a system of 2 modular congruences
- Using the Chinese Remainder Theorem to compute an integer $\pmod{m}$, where m is composite

(SMT-2021-General-22) Jane is trying to create a list of all the students of a high school. When she organizes the students into 5, 7, 9, or 13 columns, there are 1, 4, 5, and 10 students left over, respectively. What is the least number of students that could be attending this school?

Chapter 12: Diophantine Equations

- Linear Diophantine equations ($ax + by = c$), Bézout's Identity
- Postage Stamp Theorem
- Sum of Squares Theorem
- Higher-degree Diophantine equations

(AMC10-2015-B15) The town of Hamlet has 3 people for each horse, 4 sheep for each cow, and 3 ducks for each person. Which of the following could not possibly be the total number of people, horses, sheep, cows, and ducks in Hamlet?