Topics & Sample Problems

MC25F (AMC 8/MathCounts Advanced Fundamentals)

Part-II

MC25F-2 Geometry

Chapter 1: Angles

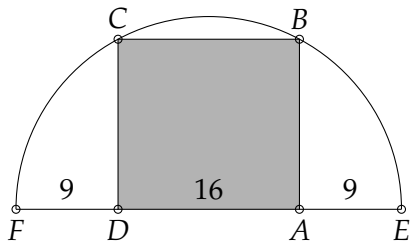
- Angle, triangle, polygon definitions, Alternate Interior Angle Theorem
- Sum of the interior and exterior angle measures in a triangle, n-sided polygon
- Inscribed Angle Theorem

(AMC8-2016-23) Two congruent circles centered at points A and B each pass through the other circle's center. The line containing both A and B is extended to intersect the circles at points C and D. The circles intersect at two points, one of which is E. What is the degree measure of $\angle CED$?

Chapter 2: Pythagorean Theorem, Special Triangles

- Pythagorean Theorem
- Special right triangles (45-45-90, 30-60-90)
- Pythagorean triples (e.g., $(3, 4, 5)$, $(5, 12, 13)$, $(8, 15, 17)$)

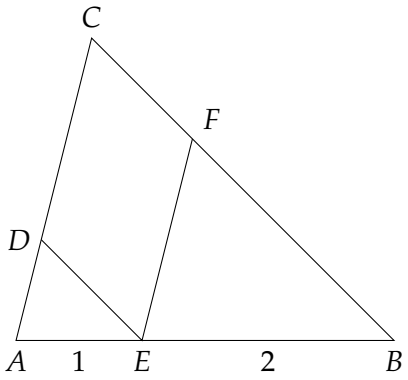
(AMC8-2020-18) Rectangle $ABCD$ is inscribed in a semicircle with diameter $\overline{FE}$, as shown in the figure. Let $DA = 16$, and let $FD = AE = 9$. What is the area of $ABCD$?



Chapter 3: Similarity

- Similarity, congruence axioms (SSS, SAS, ASA, AA, HL)
- Angle Bisector Theorem

(AMC8-2018-20) In $\triangle ABC$, a point E is on $\overline{AB}$ with $AE = 1$ and $EB = 2$. Point D is on $\overline{AC}$ so that $\overline{DE} \parallel \overline{BC}$ and point F is on $\overline{BC}$ so that $\overline{EF} \parallel \overline{AC}$. What is the ratio of the area of $CDEF$ to the area of $\triangle ABC$?



Chapter 4: Length-1

- Perimeter of a polygon, Triangle Inequality
- Solving more advanced geometry problems with the Pythagorean Theorem
- Determining if a triangle is right, acute, or obtuse given its side lengths

(MathCounts-2014-Chapter-Sprint-29) Point E lies within rectangle $ABCD$. If $AE = 7$, $BE = 5$ and $CE = 8$, what is DE? Express your answer in simplest radical form.

Chapter 5: Length-2

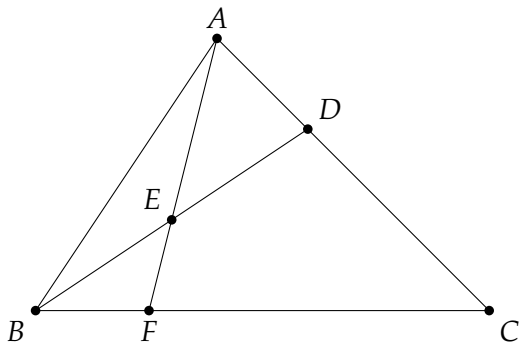
- Circumference of a circle, definition of π
- Power of a Point
- Inscribed and circumscribed circles of a triangle
- Ravi substitution

(BmMT-2013-Individual-19) Circles of radii 20 and 13 are externally tangent at T. The common external tangent touches the circles at A and B, respectively, where $A \neq B$. The common internal tangent of the circles at T intersects segment AB at X. Find the length of AX.

Chapter 6: Length-3

- Mass points technique, levers, torque
- Ceva's and Menelaus' Theorems

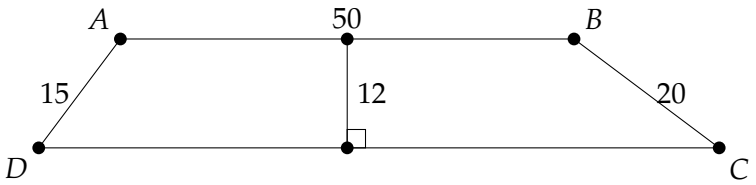
(AMC8-2019-24) In triangle ABC, point D divides side $\overline{AC}$ so that $AD : DC = 1 : 2$. Let E be the midpoint of $\overline{BD}$ and let F be the point of intersection of line BC and line AE. Given that the area of $\triangle ABC$ is 360, what is the area of $\triangle EBF$?



Chapter 7: Area-1

- Area formulas for polygons (triangles, rectangles, trapezoids, kites, etc.)
- Triangle area formulas (Heron's formula, $A = rs$, $A = \frac{abc}{4R}$)

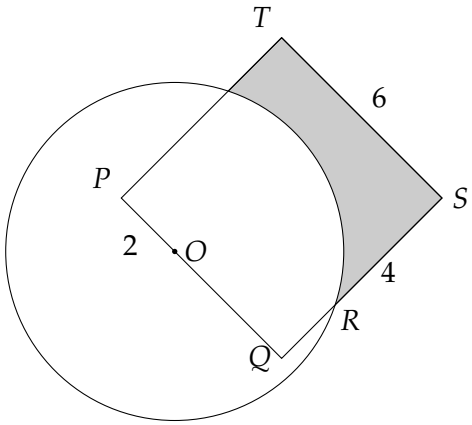
(AMC8-2011-20) Quadrilateral $ABCD$ is a trapezoid, $AD = 15$, $AB = 50$, $BC = 20$, and the altitude is 12. What is the area of the trapezoid?



Chapter 8: Area-2

- Area formulas for circles, sectors

(MathCounts-2023-State-Target-4) In the figure shown, the center of circle O is located on side PQ of square $PQST$. Given that $PO = 2$ cm, $RS = 4$ cm, and $ST = 6$ cm, what is the area of the shaded region? Express your answer in terms of π.



Chapter 9: Analytic Geometry-1

- Cartesian coordinate plane
- Graphing lines in the coordinate plane, defn. of slope
- Midpoint and distance formulas
- Distance from a point to a line

(AMC8-2019-21) What is the area of the triangle formed by the lines $y = 5$, $y = 1 + x$, and $y = 1 - x$?

Chapter 10: Analytic Geometry-2

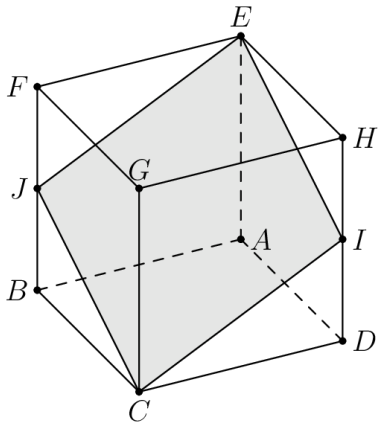
- Coordinate transformations (reflection, rotation, etc.)
- Equation of a circle in the coordinate plane
- Shoelace Formula

(MathCounts-2019-State-Sprint-28) The circles given by the equations $x^2 + y^2 = 169$ and $x^2 + (y - 14)^2 = 225$ have a common chord. How many units long is that chord?

Chapter 11: 3D-1

- Distance formula in 3 dimensions
- Defn. of polyhedron, types of polyhedra
- Surface area of various 3-dimensional shapes (prisms, cones, spheres)

(AMC8-2018-24) In the cube $ABCDEFGH$ with opposite vertices C and E, J and I are the midpoints of edges $\overline{FB}$ and $\overline{HD}$, respectively. Let R be the ratio of the area of the cross-section $EJCI$ to the area of one of the faces of the cube. What is R^2?



Chapter 12: 3D-2

- Euler's Polyhedral Formula
- Volume of various 3-dimensional shapes

(AMC12-2021-Spring-B6) An inverted cone with base radius 12 cm and height 18 cm is full of water. The water is poured into a tall cylinder whose horizontal base has radius of 24 cm. What is the height in centimeters of the water in the cylinder?

MC25F-2 Number Theory

Chapter 1: Gauss Sums

- Sums of arithmetic sequences (e.g., sum of the first n positive integers)
- Sum of Squares, Sum of Cubes formulas

(AMC12-2010-A2) A ferry boat shuttles tourists to an island every hour starting at 10 AM until its last trip, which starts at 3 PM. One day the boat captain notes that on the 10 AM trip there were 100 tourists on the ferry boat, and that on each successive trip, the number of tourists was 1 fewer than on the previous trip. How many tourists did the ferry take to the island that day?

Chapter 2: Primes & Prime Factorization

- Defn. of factor, proper factor, multiple
- Defn. of prime and composite
- Sieve of Eratosthenes, determining if a number is prime or composite
- Fundamental Theorem of Arithmetic
- Legendre's Formula

(AMC8-2017-19) What is the largest integer n for which 5^n is a factor of the sum $98! + 99! + 100!$?

Chapter 3: Divisibility

- Divisibility rules for the integers 2 through 11, inclusive
- Determining if an integer is divisible by a larger number (e.g., 36 or 99) using these divisibility rules

(MathCounts-2023-State-Team-3) A six-digit number is written as 839A6B where A and B are two unknown digits. If the number is divisible by 22 and 4, what is the greatest possible value of $|A - B|$?

Chapter 4: Number of Divisors

- Review of the Multiplication Principle
- Using the Multiplication Principle to determine $d(n)$, the number of positive divisors of n, given its prime factorization
- Defn. of multiplicative function, recognizing that $d(n)$ is multiplicative
- Determining integers which have a given number of divisors

(MathCounts-2023-State-Countdown-46) What is the least positive integer with 8 odd positive divisors and 16 even positive divisors?

Chapter 5: Sum of Divisors

- Definition of $\sigma(n)$, the sum of the divisors of n
- Determining the sum of divisors of n using the prime factorization of n
- Recognizing that $\sigma(n)$ is multiplicative

(AMC10-2021-Spring-B12) Let $N = 34 \cdot 34 \cdot 63 \cdot 270$. What is the ratio of the sum of the odd divisors of N to the sum of the even divisors of N?

Chapter 6: Factoring Techniques

- Difference of Squares $(a^2 - b^2 = (a - b)(a + b))$
- Sum and Difference of Cubes $(a^3 \pm b^3)$
- Simon's Favorite Factoring Trick

(BmMT-2017-Team-9) How many nonnegative integer pairs (a, b) are there that satisfy $ab = 90 - a - b$?

Chapter 7: Number Bases

- Base 10 and base b
- Defn. of the base-b representation of a number n
- Converting integers between different number bases
- Arithmetic in different bases
- Decimal and fractional bases
- Fast base conversion (e.g., base 2 to base 16)

(MathCounts-2018-State-Sprint-22) The base-four representation of $p = 3 + \frac{0}{4} + \frac{2}{4^2} + \frac{1}{4^3}$ is $p = 3.021_4$. In base eight, $p = 3.AB_8$. What is the value of $A + B$?

Chapter 8: GCD & LCM

- Defn. of greatest common divisor, least common multiple, relatively prime
- Properties of GCD and LCM
- Euclidean Algorithm

(AMC10-2022-A7) The least common multiple of a positive integer n and 18 is 180, and the greatest common divisor of n and 45 is 15. What is the sum of the digits of n?

Chapter 9: Modular Arithmetic

- Defn. of congruence ($a \equiv b \pmod{m}$), basic properties of the $\equiv$ relation
- Finding remainders $\pmod{m}$ using patterns or properties of modular arithmetic
- Multiplicative inverse $a^{-1} \pmod{m}$
- Using modular arithmetic to find the remainder when an integer is divided by 9 or 11

(AMC8-2011-22) What is the tens digit of 7^{2011}?

Chapter 10: FLT and Euler's Totient Theorem

- Fermat's Little Theorem ($a^{p-1} \equiv 1 \pmod{p}$)
- Introduction to $\varphi(n)$ and Euler's Totient Theorem

(MathCounts-2022-State-Sprint-18) How many integers between 1 and 280, inclusive, are not divisible by 2, 5 or 7?

Chapter 11: Chinese Remainder Theorem

- Chinese remainder for a system of 2 modular congruences, and k modular congruences
- Solving a system of modular congruences via guess-and-check
- Using the Chinese Remainder Theorem to compute an integer $\pmod{m}$, where m is composite

(BmMT-2018-Speed-30) If the chairs in an auditorium are organized into rows of 13 chairs, there are 11 left over. If they are organized into rows of 7 chairs, there are 5 left over. What is the smallest number of chairs that the auditorium could have?

Chapter 12: Diophantine Equations

- Linear Diophantine equations ($ax + by = c$), Bézout's Identity
- Postage Stamp Theorem
- Simple higher-degree Diophantine equations

(BmMT-2014-Team-9) What is the ordered pair (x, y) of positive integers such that $144x - 89y = 1$ and x is minimal?