

Topics & Sample Problems

MC25F (AMC 8/MathCounts Advanced Fundamentals)



Part-I

MC25F-1 Algebra

Each chapter is followed by a short summary of topics covered in that chapter, along with one sample contest problem from that chapter.

Chapter 1: Integers & Arithmetic

- Integers, types of integers
- Order of operations (PEMDAS)
- Variables, solving linear equations in terms of one variable
- Translating word problems into equations involving one variable

(AMC8-2024-21) A group of frogs (called an army) is living in a tree. A frog turns green when in the shade and turns yellow when in the sun. Initially, the ratio of green to yellow frogs was 3 : 1. Then 3 green frogs moved to the sunny side and 5 yellow frogs moved to the shady side. Now the ratio is 4 : 1. What is the difference between the number of green frogs and the number of yellow frogs now?

Chapter 2: Fractions & Decimals

- Arithmetic with fractions and decimals
- Types of fractions (proper, improper, mixed number), defn. of rational number
- Cross-multiplying
- Converting fractions to decimals and vice versa
- Telescoping sums and products

(AMC10-2021-Spring-A8) When a student multiplied the number 66 by the repeating decimal,

$$1.\underline{a}\underline{b}\underline{a}\underline{b}\dots = 1.\overline{a\,b},$$

where a and b are digits, he did not notice the notation and just multiplied 66 times $1.\underline{a}\underline{b}$. Later he found that his answer is 0.5 less than the correct answer. What is the 2-digit number $\underline{a}\underline{b}$?

Chapter 3: Percent

- Word problems involving percentages (e.g., tax, tip, discounts)
- Calculating interest

(AMC10-2008-A8) Heather compares the price of a new computer at two different stores. Store A offers 15% off the sticker price followed by a \$90 rebate, and store B offers 25% off the same sticker price with no rebate. Heather saves \$15 by buying the computer at store A instead of store B. What is the sticker price of the computer, in dollars?

Chapter 4: Exponents

- Definition, properties of exponentiation
- Negative exponents (e.g., 4^{-3})
- Word problems with exponents

(MathCounts-2022-State-Sprint-26) If $(7^2 + 24^2)^4 \times (5^2 + 10^2)^5 \times (75^2 + 100^2)^6 = 5^n$, what is the value of n ?

Chapter 5: Radicals

- Defn. of square root, n^{th} root
- Fractional and real exponents
- Converting a radical expression (e.g., $\sqrt{12}$ into simplest radical form
- Rationalizing the denominator

(MathCounts-2023-Chapter-Sprint-26) What is the value of $\frac{\sqrt{45} + 2\sqrt{15}}{\sqrt{128} + 2\sqrt{24}}$? Express your answer as a common fraction in simplest radical form.

Chapter 6: Systems of Equations

- Solving a system of two linear equations using substitution, elimination, or graphing
- Determining whether a system of linear equations has a solution
- Solving a simple system of two or more non-linear equations
- Converting a word problem into a system of equations

(AMC10-2015-A8) Two years ago Pete was three times as old as his cousin Claire. 2 years before that, Pete was four times as old as Claire. In how many years will the ratio of their ages be 2:1?

Chapter 7: Distance, Rate, and Time

- Unit conversions
- Distance equals rate times time ($d = rt$)
- Average speed, relative speed
- Defn. of harmonic mean
- Work/output problems involving $d = rt$

(AMC10-2021-Spring-A6) Chantal and Jean start hiking from a trailhead toward a fire tower. Jean is wearing a heavy backpack and walks slower. Chantal starts walking at 4 miles per hour. Halfway to the tower, the trail becomes really steep, and Chantal slows down to 2 miles per hour. After reaching the tower, she immediately turns around and descends the steep part of the trail at 3 miles per hour. She meets Jean at the halfway point. What was Jean's average speed, in miles per hour, until they meet?

Chapter 8: Statistics

- Defn. of mode, range, median, arithmetic mean, geometric mean
- Weighted average
- Analyzing charts (e.g., bar charts, histograms, line graphs)

(AMC8-2012-22) Let R be a set of nine distinct integers. Six of the elements of the set are 2, 3, 4, 6, 9, and 14. What is the number of possible values of the median of R ?

Chapter 9: Sequences and Series

- Defn. of sequence and series
- Arithmetic and geometric sequences, series
- Recursive sequences (e.g., Fibonacci sequence F_n)

(MathCounts-2023-State-Countdown-44) On January 1, Jane walks 27 miles. On January 2, Jane walks 9 miles. On January 3, Jane walks 3 miles. She continues in this way, so that each day she walks $\frac{1}{3}$ as far as the day before. There are 31 days in January. How many miles does Jane walk in January? Express your answer as a decimal to the nearest tenth.

Chapter 10: Functions & Operations

- Function definitions (domain, codomain, range, etc.)
- Inverse functions, piecewise functions
- Operators, absolute value, floor/ceiling value
- Graphing functions in the xy -plane
- Solving simple functional equations

(AMC8-2017-21) Suppose a , b , and c are nonzero real numbers, and $a + b + c = 0$. What are the possible value(s) for $\frac{a}{|a|} + \frac{b}{|b|} + \frac{c}{|c|} + \frac{abc}{|abc|}$?

Chapter 11: Polynomials-1

- Polynomials of a single variable
- Defn. of degree, leading coefficient, root
- Quadratic polynomials
- Solving for the roots of a quadratic polynomial by completing the square, factoring, or the Quadratic Formula
- Discriminant of a quadratic polynomial, vertex of a parabola

(AMC10-2020-A5) What is the sum of all real numbers x for which $|x^2 - 12x + 34| = 2$?

Chapter 12: Polynomials-2

- Fundamental Theorem of Algebra, Factor Theorem
- Rational Root Theorem
- Vieta's Formulas for quadratic, cubic, and higher-degree polynomials

(MathCounts-2018-State-Countdown-50) If $3x^3 - 93x^2 + 933x - 3003 = 0$ can be factored as $3(x - a)(x - b)(x - c) = 0$ for some numbers a , b , and c , what is the surface area, in square units, of a rectangular prism with edge lengths a , b , and c ?

MC25F-1 Counting

Each chapter is followed by a short summary of topics covered in that chapter, along with one sample contest problem from that chapter.

Chapter 1: Addition/Multiplication Principles

- Solving simple counting (enumeration) problems using the Addition and/or Multiplication Principles
- Visualizing the Multiplication Principle using a decision tree

(AMC8-2009-22) How many whole numbers between 1 and 1000 do **not** contain the digit 1?

Chapter 2: Permutations

- Defn. of permutation, factorial (e.g., $n!$)
- Partial permutations, formula for $P(n, k)$

(AMC8-2018-16) Professor Chang has nine different language books lined up on a bookshelf: two Arabic, three German, and four Spanish. How many ways are there to arrange the nine books on the shelf keeping the Arabic books together and keeping the Spanish books together?

Chapter 3: Combinations

- Combinations vs. partial permutations
- Binomial coefficients, formula for $\binom{n}{k}$
- The identity $\binom{n}{k} = \binom{n}{n-k}$

(MathCounts-2022-State-Sprint-24) Thomas, Carrie and Lenny each captain a different one of three hockey teams. Each captain will choose four players from a pool of 12 players, with each player chosen for only one team. How many different ways can the teams be formed?

Chapter 4: Casework

- Solving a counting problem by breaking it into simpler, disjoint cases
- Review, practice with counting topics from Chapters 1-3 using casework

(AMC8-2022-23) A \triangle or \bigcirc is placed in each of the nine squares in a 3-by-3 grid. Shown below is a sample configuration with three \triangle s in a line.

\triangle	\bigcirc	\triangle
\triangle	\bigcirc	\triangle
\triangle	\triangle	\bigcirc

How many configurations will have three \triangle s in a line and three \bigcirc s in a line?

Chapter 5: Complementary Counting & Overcounting

- Solving a counting problem using either complementary counting or overcounting
- Number of permutations with repeated elements (e.g., letters in the word BANANA)
- Introduction to multinomial coefficients $\binom{n}{a_1, a_2, \dots, a_k}$
- Common overcounting problems (e.g., number of seatings at a round table)

(MathCounts-2014-State-Sprint-26) Seven different prizes are to be distributed among three contest winners such that each winner receives at least one prize and each of the prizes goes to one of the three winners. In how many different ways can the prizes be distributed among the three winners?

Chapter 6: Counting Sets

- Set definitions and notation (e.g., \in , \subseteq , \subset , \cup , \cap , $|A|$)
- Subsets, number of subsets of a set
- Principle of Inclusion-Exclusion for 2 or 3 sets

(AMC8-2010-20) In a room, $\frac{2}{5}$ of the people are wearing gloves, and $\frac{3}{4}$ of the people are wearing hats. What is the minimum number of people in the room wearing both a hat and a glove?

Chapter 7: Counting Shapes & Paths

- Number of paths in a lattice grid
- Counting the number of paths or shapes efficiently using combinations or other methods
- Counting with recursion

(MathCounts-2020-Chapter-Sprint-26) A rectangular prism of size $3 \times 4 \times 5$ is made from 60 unit cubes. Including the full $3 \times 4 \times 5$ prism and the 60 unit prisms, how many total rectangular prisms can be found in the large rectangular prism by taking a subset of the 60 cubes?

Chapter 8: Counting with Digits

- Solving problems involving finding the number of n -digit integers satisfying some property
- Palindromes, numbers with increasing or decreasing digits

(AMC8-2011-23) How many 4-digit positive integers have four different digits, where the leading digit is not zero, the integer is a multiple of 5, and 5 is the largest digit?

Chapter 9: Stars and Bars

- Stars and Bars derivation, formulas (both positive and non-negative variants)
- Solving counting problems using the Stars and Bars technique

(AMC8-2019-25) Alice has 24 apples. In how many ways can she share them with Becky and Chris so that each of the three people has at least two apples?

Chapter 10: Binomial & Pascal's Triangle

- Expansion of $(x + y)^n$, Binomial Theorem, Pascal's triangle
- Basic combinatorial identities (e.g., Pascal's Identity, Hockey Stick Identity)
- Introduction to the Multinomial Theorem

(BmMT-2015-Speed-31) In the song "Twelve Days of Christmas", you get many gifts over a period of 12 days. On day n , you get $n + (n - 1) + (n - 2) + \dots + 1$ gifts to be specific. How many gifts do you get in total?

Chapter 11: Probability-1

- Probability definitions (definition of $P(E)$, sample space, independent or disjoint events)
- Binomial distribution ($P(k \text{ out of } n \text{ successes})$)
- Recursive probability

(AMC8-2016-21) A top hat contains 3 red chips and 2 green chips. Chips are drawn randomly one at a time without replacement until all 3 of the reds are drawn or until both green chips are drawn. What is the probability that all 3 reds are drawn?

Chapter 12: Probability-2

- Geometric probability in 2 and 3 dimensions
- Expected value, linearity of expectation
- Conditional probability, Bayes' Theorem

(MathCounts-2023-Chapter-Team-9) Pete rolls three fair six-sided dice. Given that the sum of the three numbers rolled is 9, what is the probability that all the dice showed different numbers? Express your answer as a common fraction.