

Topics & Sample Problems

MC20F (AMC 8/MathCounts Basic Fundamentals)



Part-II

MC20F-2 Geometry

Chapter 1: Angles

- Angle, triangle, polygon definitions, Alternate Interior Angle Theorem
- Sum of the interior and exterior angle measures in a triangle, n -sided polygon
- Inscribed Angle Theorem

(MathCounts-2020-Chapter-Sprint-7) In quadrilateral $ABCD$, $m\angle A = 119^\circ$, $m\angle B = 89^\circ$ and $m\angle C = 49^\circ$. What is the degree measure of $\angle D$?

Chapter 2: Pythagorean Theorem, Special Triangles

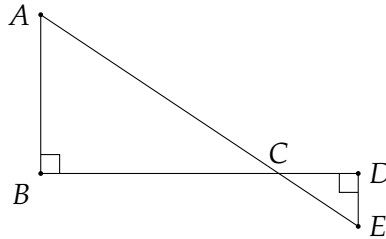
- Pythagorean Theorem
- Special right triangles (45-45-90, 30-60-90)
- Pythagorean triples (e.g., $(3, 4, 5)$, $(5, 12, 13)$, $(8, 15, 17)$)

(AMC8-2005-7) Bill walks $\frac{1}{2}$ mile south, then $\frac{3}{4}$ mile east, and finally $\frac{1}{2}$ mile south. How many miles is he, in a direct line, from his starting point?

Chapter 3: Similarity

- Similarity, congruence axioms (SSS, SAS, ASA, AA, HL)
- Angle Bisector Theorem

(CEMC-2011-Gauss8-15) In the diagram, AE and BD are straight lines that intersect at C . If $BD = 16$, $AB = 9$, $CE = 5$, and $DE = 3$, then the length of AC is



Chapter 4: Length-1

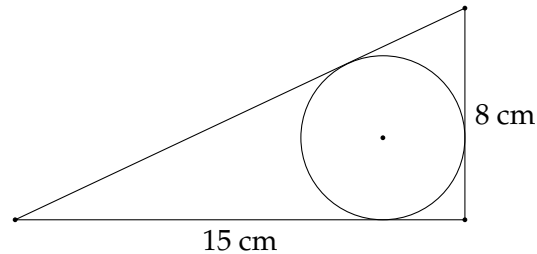
- Perimeter of a polygon, Triangle Inequality
- Solving more advanced geometry problems with the Pythagorean Theorem
- Determining if a triangle is right, acute, or obtuse given its side lengths

(AMC8-2015-8) What is the smallest whole number larger than the perimeter of any triangle with a side of length 5 and a side of length 19?

Chapter 5: Length-2

- Circumference of a circle, definition of π
- Power of a Point
- Inscribed and circumscribed circles of a triangle
- Ravi substitution

(UNB-2010-Gr 9-26) A circle is inscribed in a right triangle. The legs of the triangle are 8 cm and 15 cm, as shown. What is the radius of the circle?



Chapter 6: Length-3

- Mass points technique, levers, torque
- Ceva's and Menelaus' Theorems

(Lehigh MC-2017-16) In triangle ABC , point F lies on AC with the ratio $AF : FC = 2 : 3$. Point D is the midpoint of BF , and AD is extended to meet BC at point E . What is the ratio $BE : EC$? Express your answer as a common fraction in reduced form.

Chapter 7: Area-1

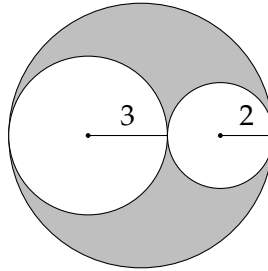
- Area formulas for polygons (triangles, rectangles, trapezoids, kites, etc.)
- Triangle area formulas (Heron's formula, $A = rs$, $A = \frac{abc}{4R}$)

(AMC8-2015-6) In $\triangle ABC$, $AB = BC = 29$, and $AC = 42$. What is the area of $\triangle ABC$?

Chapter 8: Area-2

- Area formulas for circles, sectors

(AMC10-2002-B5) Circles of radius 2 and 3 are externally tangent and are circumscribed by a third circle, as shown in the figure. Find the area of the shaded region.



Chapter 9: Analytic Geometry-1

- Cartesian coordinate plane
- Graphing lines in the coordinate plane, defn. of slope
- Midpoint and distance formulas
- Distance from a point to a line

(BmMT-2012-Individual-1) What is the slope of the line perpendicular to the graph $\frac{x}{4} + \frac{y}{9} = 1$ at $(0, 9)$?

Chapter 10: Analytic Geometry-2

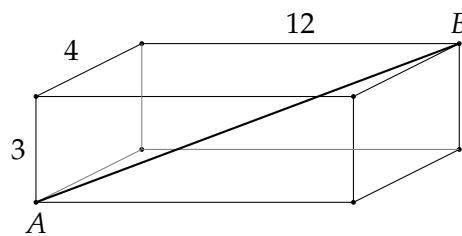
- Coordinate transformations (reflection, rotation, etc.)
- Equation of a circle in the coordinate plane
- Shoelace Formula

(AMC12-2022-B5) The point $(-1, -2)$ is rotated 270° counterclockwise about the point $(3, 1)$. What are the coordinates of its new position?

Chapter 11: 3D-1

- Distance formula in 3 dimensions
- Defn. of polyhedron, types of polyhedra
- Surface area of various 3-dimensional shapes (prisms, cones, spheres)

(UNB-2017-Gr 9-12) A 12 cm by 4 cm by 3 cm rectangular box is shown below. What is the length, in centimeters, of the diagonal AB ?



Chapter 12: 3D-2

- Euler's Polyhedral Formula
- Volume of various 3-dimensional shapes

(AMC8-2019-9) Alex and Felicia each have cats as pets. Alex buys cat food in cylindrical cans that are 6 cm in diameter and 12 cm high. Felicia buys cat food in cylindrical cans that are 12 cm in diameter and 6 cm high. What is the ratio of the volume of one of Alex's cans to the volume one of Felicia's cans?

MC20F-2 Number Theory

Chapter 1: Gauss Sums

- Sums of arithmetic sequences (e.g., sum of the first n positive integers)
- Sum of Squares, Sum of Cubes formulas

(MathCounts-2014-Chapter-Sprint-13) What is the sum of the first 2000 positive integers?

Chapter 2: Primes & Prime Factorization

- Defn. of factor, proper factor, multiple
- Defn. of prime and composite
- Sieve of Eratosthenes, determining if a number is prime or composite
- Fundamental Theorem of Arithmetic
- Legendre's Formula

(AMC8-2011-17) Let w , x , y , and z be whole numbers. If $2^w \cdot 3^x \cdot 5^y \cdot 7^z = 588$, then what does $2w + 3x + 5y + 7z$ equal?

Chapter 3: Divisibility

- Divisibility rules for the integers 2 through 11, inclusive
- Determining if an integer is divisible by a larger number (e.g., 36 or 99) using these divisibility rules

(AMC8-2018-7) The 5-digit number $\underline{2} \underline{0} \underline{1} \underline{8} \underline{U}$ is divisible by 9. What is the remainder when this number is divided by 8?

Chapter 4: Number of Divisors

- Review of the Multiplication Principle
- Using the Multiplication Principle to determine $d(n)$, the number of positive divisors of n , given its prime factorization
- Defn. of multiplicative function, recognizing that $d(n)$ is multiplicative
- Determining integers which have a given number of divisors

(BmMT-2018-Individual-8) How many integers n with $1 \leq n \leq 100$ have exactly 3 positive divisors?

Chapter 5: Sum of Divisors

- Definition of $\sigma(n)$, the sum of the divisors of n
- Determining the sum of divisors of n using the prime factorization of n
- Recognizing that $\sigma(n)$ is multiplicative

(MathCounts-2023-Chapter-Countdown-57) What is the sum of all positive integer factors of 150?

Chapter 6: Factoring Techniques

- Difference of Squares ($a^2 - b^2 = (a - b)(a + b)$)
- Sum and Difference of Cubes ($a^3 \pm b^3$)
- Simon's Favorite Factoring Trick

(MathCounts-2023-State-Countdown-31) What is the value of the expression $2023^2 - 1977^2$?

Chapter 7: Number Bases

- Base 10 and base b
- Defn. of the base- b representation of a number n
- Converting integers between different number bases
- Arithmetic in different bases
- Decimal and fractional bases
- Fast base conversion (e.g., base 2 to base 16)

(MathCounts-2014-State-Sprint-20) In base 5, what is the value of $27_{10} \times 314_5$?

Chapter 8: GCD & LCM

- Defn. of greatest common divisor, least common multiple, relatively prime
- Properties of GCD and LCM
- Euclidean Algorithm

(AMC8-2013-10) What is the ratio of the least common multiple of 180 and 594 to the greatest common factor of 180 and 594?

Chapter 9: Modular Arithmetic

- Defn. of congruence ($a \equiv b \pmod{m}$), basic properties of the $\equiv$ relation
- Finding remainders $\pmod{m}$ using patterns or properties of modular arithmetic
- Multiplicative inverse $a^{-1} \pmod{m}$
- Using modular arithmetic to find the remainder when an integer is divided by 9 or 11

(AMC8-2000-14) What is the units digit of $19^{19} + 99^{99}$?

Chapter 10: FLT and Euler's Totient Theorem

- Fermat's Little Theorem ($a^{p-1} \equiv 1 \pmod{p}$)
- Introduction to $\varphi(n)$ and Euler's Totient Theorem

(BmMT-2012-Ciphering-31) What is the remainder when 19^{19} is divided by 17?

Chapter 11: Chinese Remainder Theorem

- Chinese remainder for a system of 2 modular congruences, and k modular congruences
- Solving a system of modular congruences via guess-and-check
- Using the Chinese Remainder Theorem to compute an integer $(\text{mod } m)$, where m is composite

(MathCounts-2019-State-Countdown-28) When Christine arranges her pennies in rows of 9, she has 8 pennies left over. When she arranges the same set of pennies in rows of 11, she has 1 penny left over. What is the smallest number of pennies Christine could have?

Chapter 12: Diophantine Equations

- Linear Diophantine equations ($ax + by = c$), Bézout's Identity
- Postage Stamp Theorem
- Simple higher-degree Diophantine equations

(MathCounts-2023-Chapter-Countdown-75) Flowers can be purchased in bundles of either 6 or 13. What is the greatest number of flowers that cannot be purchased?