Topics & Sample Problems

MC20F (AMC 8/MathCounts Basic Fundamentals)



Part-I

MC20F-1 Algebra

Each chapter is followed by a short summary of topics covered in that chapter, along with one sample contest problem from that chapter.

Chapter 1: Integers & Arithmetic

- Integers, types of integers
- Order of operations (PEMDAS)
- Variables, solving linear equations in terms of one variable
- Translating word problems into equations involving one variable

(MathCounts-2020-Chapter-Sprint-18) For a charity walk-a-thon, Jen donated \$10 and pledged to donate 10¢ for each lap Mira walked. Joy pledged to donate 35¢ for each lap Mira walked. If the total amount Jen donated equals the total amount Joy donated, how many laps did Mira walk?

Chapter 2: Fractions & Decimals

- Arithmetic with fractions and decimals
- Types of fractions (proper, improper, mixed number), defn. of rational number
- Cross-multiplying
- Converting fractions to decimals and vice versa
- Telescoping sums and products

(AMC8-2018-10) The *harmonic mean* of a set of non-zero numbers is the reciprocal of the average of the reciprocals of the numbers. What is the harmonic mean of 1, 2, and 4?



Chapter 3: Percent

- Word problems involving percentages (e.g., tax, tip, discounts)
- Calculating interest

(CEMC-2000-Gauss7-13) Karl had his salary reduced by 10%. He was later promoted and his salary was increased by 10%. If his original salary was \$20,000, what is his present salary?

Chapter 4: Exponents

- Definition, properties of exponentiation
- Negative exponents (e.g., 4⁻³)
- Word problems with exponents

(BmMT-2012-Team-2) How many digits does the product $2^{42} \cdot 5^{38}$ have?

Chapter 5: Radicals

- Defn. of square root, n^{th} root
- Fractional and real exponents
- Converting a radical expression (e.g., $\sqrt{12}$ into simplest radical form
- Rationalizing the denominator

(MathCounts-2023-Chapter-Sprint-11) What is the value of $\sqrt{\frac{1}{4}} + \sqrt{\frac{1}{9}} + \sqrt{\frac{1}{16}}$? Express your answer as a common fraction.

Chapter 6: Systems of Equations

- Solving a system of two linear equations using substitution, elimination, or graphing
- Determining whether a system of linear equations has a solution
- Solving a simple system of two or more non-linear equations
- Converting a word problem into a system of equations



(AMC8-2012-9) The Fort Worth Zoo has a number of two-legged birds and a number of fourlegged mammals. On one visit to the zoo, Margie counted 200 heads and 522 legs. How many of the animals that Margie counted were two-legged birds?

Chapter 7: Distance, Rate, and Time

- Unit conversions
- Distance equals rate times time (d = rt)
- Average speed, relative speed
- Defn. of harmonic mean
- Work/output problems involving *d* = *rt*

(AMC8-2009-14) Austin and Temple are 50 miles apart along Interstate 35. Bonnie drove from Austin to her daughter's house in Temple, averaging 60 miles per hour. Leaving the car with her daughter, Bonnie rode a bus back to Austin along the same route and averaged 40 miles per hour on the return trip. What was the average speed for the round trip, in miles per hour?

Chapter 8: Statistics

- Defn. of mode, range, median, arithmetic mean, geometric mean
- Weighted average
- Analyzing charts (e.g., bar charts, histograms, line graphs)

(AMC8-2008-10) The average age of the 6 people in Room A is 40. The average age of the 4 people in Room B is 25. If the two groups are combined, what is the average age of all the people?

Chapter 9: Sequences and Series

- Defn. of sequence and series
- Arithmetic and geometric sequences, series
- Recursive sequences (e.g., Fibonacci sequence *F_n*)

(AMC8-2008-12) A ball is dropped from a height of 3 meters. On its first bounce it rises to a height of 2 meters. It keeps falling and bouncing to $\frac{2}{3}$ of the height it reached in the previous bounce. On which bounce will it not rise to a height of 0.5 meters?



Chapter 10: Functions & Operations

- Function definitions (domain, codomain, range, etc.)
- Inverse functions, piecewise functions
- Operators, absolute value, floor/ceiling value
- Graphing functions in the *xy*-plane
- Solving simple functional equations

(AMC8-2016-10) Suppose that a * b means 3a - b. What is the value of x if 2 * (5 * x) = 1?

Chapter 11: Polynomials-1

- Polynomials of a single variable
- Defn. of degree, leading coefficient, root
- Quadratic polynomials
- Solving for the roots of a quadratic polynomial by completing the square, factoring, or the Quadratic Formula
- Discriminant of a quadratic polynomial, vertex of a parabola

(MathCounts-2019-Chapter-Sprint-17) The two solutions of the equation $x^2 + ax + 14 = 0$ are x = 2 and x = 7. What is the value of *a*?

Chapter 12: Polynomials-2

- Fundamental Theorem of Algebra, Factor Theorem
- Rational Root Theorem
- Vieta's Formulas for quadratic, cubic, and higher-degree polynomials

(MathCounts-2018-State-Countdown-70) If the cubic equation $x^3 + ax + b = 0$ has three real solutions, x = 7, x = 5 and x = m, what is the value of m?

MC20F-1 Counting

Each chapter is followed by a short summary of topics covered in that chapter, along with one sample contest problem from that chapter.

Chapter 1: Addition/Multiplication Principles

- Solving simple counting (enumeration) problems using the Addition and/or Multiplication Principles
- Visualizing the Multiplication Principle using a decision tree

(AMC8-2007-4) A haunted house has six windows. In how many ways can Georgie the Ghost enter the house by one window and leave by a different window?

Chapter 2: Permutations

- Defn. of permutation, factorial (e.g., *n*!)
- Partial permutations, formula for P(n,k)

(AMC8-2022-14) In how many ways can the letters in **BEEKEEPER** be rearranged so that two or more **E**s do not appear together?

Chapter 3: Combinations

- Combinations vs. partial permutations
- Binomial coefficients, formula for $\binom{n}{k}$
- The identity $\binom{n}{k} = \binom{n}{n-k}$



(BmMT-2021-Team-7) Vivienne is deciding on what courses to take for Spring 2021, and she must choose from four math courses, three computer science courses, and five English courses. Vivienne decides that she will take one English course and two additional courses that are either computer science or math, and the order of the courses does not matter. How many choices does Vivienne have?

Chapter 4: Casework

- Solving a counting problem by breaking it into simpler, disjoint cases
- Review, practice with counting topics from Chapters 1-3 using casework

(AMC8-2009-16) How many 3-digit positive integers have digits whose product equals 24?

Chapter 5: Complementary Counting & Overcounting

- Solving a counting problem using either complementary counting or overcounting
- Number of permutations with repeated elements (e.g., letters in the word BANANA)
- Introduction to multinomial coefficients $\binom{n}{a_1, a_2, \dots, a_k}$
- Common overcounting problems (e.g., number of seatings at a round table)

(AMC8-2016-17) An ATM password at Fred's Bank is composed of four digits from 0 to 9, with repeated digits allowable. If no password may begin with the sequence 9, 1, 1, then how many passwords are possible?

Chapter 6: Counting Sets

- Set definitions and notation (e.g., \in , \subseteq , \subset , \cup , \cap , |A|)
- Subsets, number of subsets of a set
- Principle of Inclusion-Exclusion for 2 or 3 sets



(AMC8-2007-13) Sets *A* and *B*, shown in the Venn diagram, have the same number of elements. Their union has 2007 elements and their intersection has 1001 elements. How many elements are in *A*?



Chapter 7: Counting Shapes & Paths

- Number of paths in a lattice grid
- Counting the number of paths or shapes efficiently using combinations or other methods
- Counting with recursion

(MathCounts-2019-School-Target-3) Starting with the G on top and only moving one letter at a time to one of the two closest letters in the row beneath it, how many different paths from top to bottom spell GREAT?

Chapter 8: Counting with Digits

- Solving problems involving finding the number of *n*-digit integers satisfying some property
- Palindromes, numbers with increasing or decreasing digits

(BmMT-2016-Team-8) How many 7-digit numbers are there such that four of its digits are 0, and the rest of the digits are odd?



Chapter 9: Stars and Bars

- Stars and Bars derivation, formulas (both positive and non-negative variants)
- Solving counting problems using the Stars and Bars technique

(AMC10-2001-A19) Pat wants to buy four donuts from an ample supply of three types of donuts: glazed, chocolate, and powdered. How many different selections are possible?

Chapter 10: Binomial & Pascal's Triangle

- Expansion of $(x + y)^n$, Binomial Theorem, Pascal's triangle
- Basic combinatorial identities (e.g., Pascal's Identity, Hockey Stick Identity)
- Introduction to the Multinomial Theorem

(BmMT-2018-Speed-27) What is the value of $37^3 + 3 \cdot 37^2 \cdot 63 + 3 \cdot 37 \cdot 63^2 + 63^3$?

Chapter 11: Probability-1

- Probability definitions (definition of P(E), sample space, independent or disjoint events)
- Binomial distribution (*P*(*k* out of *n* successes))
- Recursive probability

(AMC8-2019-18) The faces of each of two fair dice are numbered 1, 2, 3, 5, 7, and 8. When the two dice are tossed, what is the probability that their sum will be an even number?

Chapter 12: Probability-2

- Geometric probability in 2 and 3 dimensions
- Expected value, linearity of expectation
- Conditional probability, Bayes' Theorem

(MathCounts-2018-School-Sprint-23) A bag contains 25 tickets, each colored either red or yellow. Red tickets are worth \$0.50, and yellow tickets are worth \$5.00. If the expected value of a ticket drawn at random from this bag is \$3.20, how many of the tickets are red?