Topics & Sample Problems

MC25F (AMC 8/MathCounts Advanced Fundamentals)

Contents

Part-I

MC25F Algebra

Chapter 1: Integers & Arithmetic

- Order of operations with Integers (PEMDAS)
- Introducing Variables
- Word problems using arithmetic with integers

Sample Problem:

(AMC10-2004-A6) Bertha has 6 daughters and no sons. Some of her daughters have 6 daughters, and the rest have none. Bertha has a total of 30 daughters and granddaughters, and no great-granddaughters. How many of Bertha's daughters and granddaughters have no daughters?

(A) 22 **(B)** 23 **(C)** 24 **(D)** 25 **(E)** 26

Chapter 2: Fractions & Decimals

- Different types of fractions (proper/improper fractions, mixed numbers, simplest form)
- Decimals with repeating/terminating digits
- Converting between fractions and decimals
- Adding, subtracting, multiplying, dividing fractions/decimals
- Telescoping sums and products
- Word problems with fractions and decimals

Sample Problem:

(AMC8-2010-21) Hui is an avid reader. She bought a copy of the best seller *Math is Beautiful*. On the first day, she read $\frac{1}{5}$ of the pages plus 12 more, and on the second day she read $\frac{1}{4}$ of the remaining pages plus 15 more. On the third day she read $\frac{1}{3}$ of the remaining pages plus 18 more. She then realizes she has 62 pages left, which she finishes the next day. How many pages are in this book?

(A) 120 **(B)** 180 **(C)** 240 **(D)** 300 **(E)** 360

Chapter 3: Percent

- Conversions between percent and fractions/decimals
- Word problems involving percent (tax, tip, interest, etc.)
- Compound Interest
- Word problems with percent

Sample Problem:

(AMC10-2008-A8) Heather compares the price of a new computer at two different stores. Store A offers 15% off the sticker price followed by a \$90 rebate, and store B offers 25% off the same sticker price with no rebate. Heather saves \$15 by buying the computer at store A instead of store B. What is the sticker price of the computer, in dollars?

(A) 750 **(B)** 900 **(C)** 1000 **(D)** 1050 **(E)** 1500

Chapter 4: Exponents

- Basic properties of exponents (multiplying, dividing, raising an exponent to another exponent)
- Negative exponents
- Word problems with exponents

Sample Problem:

(AMC8-2010-24) What is the correct ordering of the three numbers, 10^8, 5^{12}, and 2^{24}?

(A) $2^{24} < 10^8 < 5^{12}$ **(B)** $2^{24} < 5^{12} < 10^8$ **(C)** $5^{12} < 2^{24} < 10^8$
(D) $10^8 < 5^{12} < 2^{24}$ **(E)** $10^8 < 2^{24} < 5^{12}$

Chapter 5: Radicals

- Square roots, cube roots, simplest radical form
- Negative/fractional exponents
- Rationalizing the denominator, simplifying radicals

Sample Problem:

(Jennifer Zhang) What is the value of $\frac{\sqrt{63}}{\sqrt{315}+\sqrt{140}}$, expressed as a common fraction in simplest radical form?

Chapter 6: Systems of Equations

- Converting a word problem into mathematical equations
- Solving two-unknown linear equations

Sample Problem:

(UNB-2008-Gr 9-7) Marina has a bank containing only pennies and nickels. If the pennies were nickels and the nickels were pennies, she would have exactly \$1.00 more. If the total value of the money in her bank is \$1.75, how many pennies does Marina have?

(A) 25 **(B)** 30 **(C)** 40 **(D)** 50 **(E)** Not enough information

Chapter 7: Distance, Rate, and Time

- Unit conversions
- Distance $=$ Rate $\times$ Time
- Average speed, relative speed
- Problems involving the amount of work/output done

Sample Problem:

(AMC10-2008-A6) A triathlete competes in a triathlon in which the swimming, biking, and running segments are all of the same length. The triathlete swims at a rate of 3 kilometers per hour, bikes at a rate of 20 kilometers per hour, and runs at a rate of 10 kilometers per hour. Which of the following is closest to the triathlete's average speed, in kilometers per hour, for the entire race?

(A) 3 **(B)** 4 **(C)** 5 **(D)** 6 **(E)** 7

Chapter 8: Statistics

- Mean, median, mode, range
- Weighted average

Sample Problem:

(Kevin Chang) Given a list of four positive integers whose median and mean are 20 and 23, respectively, what is the smallest possible range of these integers?

Chapter 9: Sequences and Series

- Arithmetic and geometric sequences
- Geometric series (finite and infinite)
- Recursively defined sequences (e.g. the Fibonacci sequence)

Sample Problem:

(Kevin Chang) If the degree measures of the interior angles of an octagon are all integers and form an arithmetic sequence, find the sum of all possible degree measures of the smallest angle.

Chapter 10: Functions & Operations

- Definitions of function, domain, range
- Linear functions ($f(x) = ax + b$)
- Piecewise-defined functions
- Absolute value, floor/ceiling value
- Operators

Sample Problem:

(Kevin Chang) Adding 130 gallons to an empty water tank fills 52% of the tank. How many gallons does the tank contain when it is completely full?

Chapter 11: Polynomials-1

- Polynomials of a single variable
- Definitions of degree, coefficient, root

- Quadratic polynomials and the quadratic formula

Sample Problem:

(Kevin Chang) Define $a \otimes b = ab - a - b$ for real numbers a and b. What is the value of

$$(((100 \otimes 99) \otimes 98) \otimes \cdots \otimes 2) \otimes 1?$$

Chapter 12: Polynomials-2

- Sum and product of the roots of a quadratic
- Vieta's formulas for cubic and higher degree polynomials

Sample Problem:

(Ali Gurel) Let m and n be roots of the polynomial

$$x^2 - 60x + 864.$$

Find a polynomial with roots $m + 1$ and $n + 1$.

MC25F Counting

Chapter 1: Addition/Multiplication Principles

- Addition (rule of sum)
- Multiplication (rule of product)

Sample Problem:

(Richard Spence) How many positive divisors of 10! are multiples of 10?

Chapter 2: Permutations

- Factorials, permutations
- Counting the number of permutations of n objects taken k at a time

Sample Problem:

(Richard Spence) 16 students split up into four teams of four students each: the Teleporters, the Transformers, the Timebenders, and the Mindbenders. Each team then decides one student to solve the number theory problems, one student to solve the algebra problems, one student to solve the geometry problems, and one student to solve the counting problems, such that each student in each group solves the problems from one subject. In how many ways can this be done? (Leave your answer unsimplified)

Chapter 3: Combinations

- Difference between permutations and combinations
- How to compute combinations ("n choose k")

Sample Problem:

(A-Star) Two lines intersect at a point. 3 points are given on the first line and 4 points are given on the second line. None of the seven points are the intersection point of two lines. How many triangles are there whose vertices are among these seven points?

Chapter 4: Casework

- Using casework to solve a variety of counting problems that can't be computed directly
- Use casework to break difficult problems into easier pieces

Sample Problem:

(CEMC-2012-Gauss8-24) Stones are numbered 1, 2, 3, 4, 5, 6, 7, 8, 9, 10. Three groups of stones can be selected so that the sum of each group is 11. For example, one arrangement is $\{1, 10\}$, $\{2, 3, 6\}$, $\{4, 7\}$. Including the example, how many arrangements are possible?

(A) 13 **(B)** 16 **(C)** 11 **(D)** 12 **(E)** 15

Chapter 5: Complementary Counting & Overcounting

- Applying the techniques of complementary counting or overcounting to solve problems that would be difficult otherwise

Sample Problem:

(Richard Spence) How many ways can 10 students be paired into five pairs of two students each? The ordering of the students in each pair, and the ordering of the five pairs does not matter.

Chapter 6: Counting Sets

- Definitions of set, subset, size, union, and intersection
- Principle of Inclusion-Exclusion

Sample Problem:

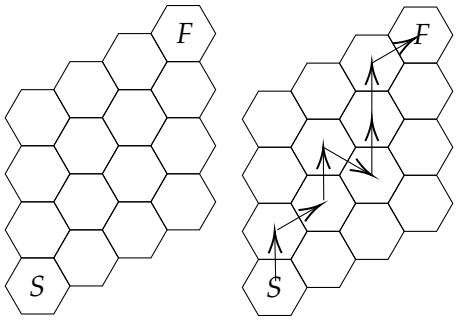
(Sean Shi) There are 150 members at a game club. 70 of them play chess, 50 play cards, 50 play checkers, 20 play chess and checkers, 14 play chess and cards, and 16 play checkers and cards. Four members play all three of these games. How many members play none of these games?

Chapter 7: Counting Shapes & Paths

- Counting the number of paths in a lattice grid using combinations and permutations
- Counting shapes or paths systematically (e.g. without counting manually)

Sample Problem:

(Richard Spence) A frog starts at S on the hexagonal grid of 16 tiles shown below. On any move, it can hop from its current hexagon to the hexagon above it, or either hexagon(s) on its right. An example sequence of moves is shown to the right. In how many different ways can the frog reach F using any such sequence of moves?



Chapter 8: Counting with Digits

- Various counting problems involving digits
- Palindromic numbers

Sample Problem:

(Kevin Chang) How many positive 3-digit integers are there whose hundreds digit equals the average of the tens and units digits?

Chapter 9: Stars and Bars

- Applying the stars and bars (or "balls and boxes") technique to solve various counting problems

Sample Problem:

(Sean Shi) How many solutions (w, x, y, z) satisfy the equation

$$w + x + y + z = 12$$

such that each variable is a non-negative integer, $x \leq 1$, and y is even?

Chapter 10: Binomial & Pascal's Triangle

- Binomial theorem (expanding $(x + y)^n$)
- Pascal's triangle

Sample Problem:

(Classic) As a closed expression in terms of n, what is the value of

$$1\binom{n}{1}+2\binom{n}{2}+3\binom{n}{3}+\cdots+n\binom{n}{n}?$$

Chapter 11: Probability-1

- Definition, of probability
- Sample space, independent/dependent events, disjoint events

Sample Problem:

(BmMT-2016-Individual-16) Alice rolls one pair of 6-sided dice and Bob rolls another pair of 6-sided dice. What is the probability that at least one of Alice's dice shows the same number as at least one of Bob's dice? Express your answer as a common fraction in reduced form.

Chapter 12: Probability-2

- Expected value and linearity of expectation
- Conditional probability, Bayes' theorem
- Geometric probability

Sample Problem:

(Richard Spence) In a certain country, exactly 1% of the population has a disease called algebritis. A certain drug test for algebritis has a 99% accuracy; i.e. it returns a correct positive (or negative) result with probability 0.99. A random participant from this country is selected and tested. Given that his algebritis test returned a positive result, what is the probability that he has algebritis? Express your answer as a common fraction in reduced form.