Topics & Sample Problems

MC20F (AMC 8/MathCounts Basic Fundamentals)

alphastar academy

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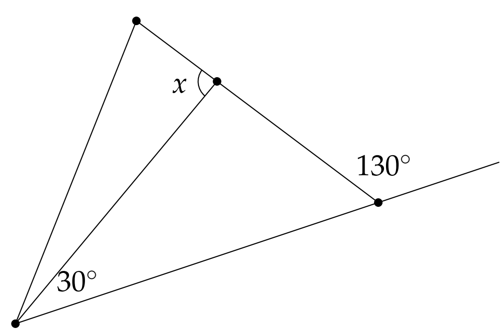
MC20F Geometry

Chapter 1: Angles

- Definitions of acute, right, obtuse, complementary, and supplementary angles
- Parallel, perpendicular, and transversal lines
- Sum of the degree measures in a triangle, different types of triangles
- Inscribed angles and arcs in circles

Sample Problem:

(Find the measure of the angle labeled x in the diagram.



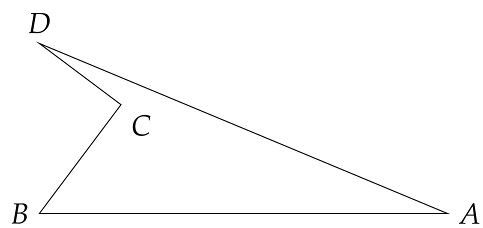
(A) 70° **(B)** 75° **(C)** 80° **(D)** 100° **(E)** 160°) C

Chapter 2: Pythagorean Theorem, Special Triangles

- 30-60-90 and 45-45-90 triangles
- Pythagorean theorem and Pythagorean triples

Sample Problem:

(In the non-convex quadrilateral $ABCD$ shown below, $\angle BCD$ is a right angle, $AB = 12$, $BC = 4$, $CD = 3$, and $AD = 13$. What is the area of quadrilateral $ABCD$?



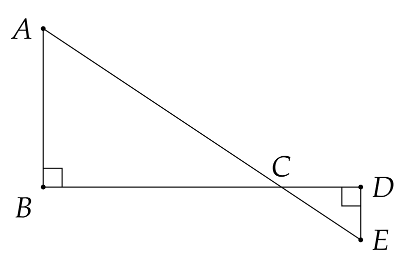
(A) 12 **(B)** 24 **(C)** 26 **(D)** 30 **(E)** 36) B

Chapter 3: Similarity

- Congruence and similarity axioms (SSS, SAS, ASA, AA)
- SSA is not a congruence axiom
- Angle bisector theorem

Sample Problem:

(In the diagram, AE and BD are straight lines that intersect at C. If $BD = 16$, $AB = 9$, $CE = 5$, and $DE = 3$, then the length of AC is



(A) 11 **(B)** 12 **(C)** 15 **(D)** 17 **(E)** 16) C

Chapter 4: Length-1

- Perimeter of polygons

- Triangle inequality
- Review of the Pythagorean theorem

Sample Problem:

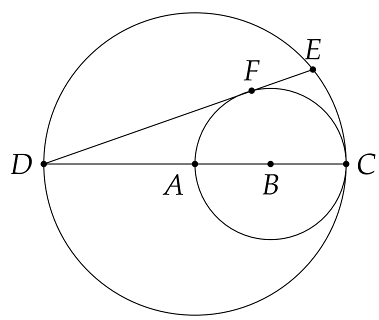
(A right triangle has legs 8 cm and 15 cm. Find the shortest altitude of this triangle, in centimeters. Express your answer as a common fraction in reduced form.) 120/17

Chapter 5: Length-2

- Circumference of a circle
- Power of a point
- Inscribed and circumscribed circles of a triangle
- Ravi substitution

Sample Problem:

(In the diagram, DC is a diameter of the larger circle centered at A, and AC is a diameter of the smaller circle centered at B. If DE is tangent to the smaller circle at F, and $DC = 12$, determine the length of DE. Express your answer as a decimal to the nearest tenth.



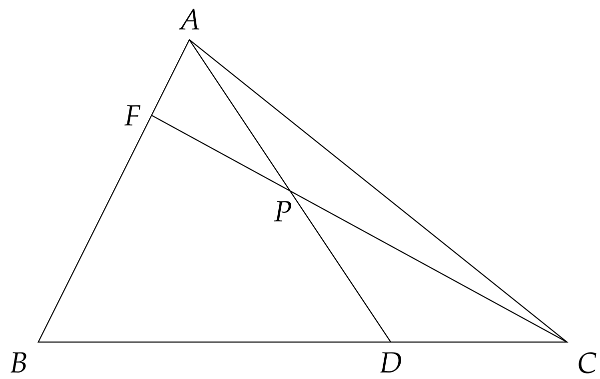
) 11.3

Chapter 6: Length-3

- Introduction to the mass points technique using physics concepts (levers, torque)
- Ceva's theorem and Menelaus' theorem

Sample Problem:

(Given $\triangle ABC$, cevians AD and CF intersect at point P inside $\triangle ABC$. Given that $AP = PD$ and $BD = 2DC$, what is $\dfrac{BF}{AF}$?



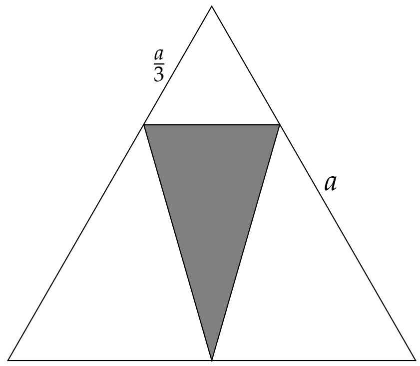
) 3

Chapter 7: Area-1

- Unit conversions (e.g. square feet to square yards)
- Areas of simple polygons (squares, rectangles, triangles, trapezoids)
- Other formulas for the area of a triangle, including Heron's

Sample Problem:

(In the figure below, the area of the shaded triangle is $2\sqrt{3}$. If the large triangle and the small upper triangle are equilateral, what is the value of a?



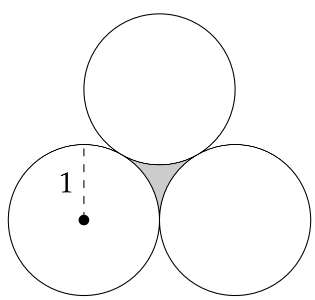
(A) 2 **(B)** 2.5 **(C)** 3 **(D)** 6 **(E)** None of these) D

Chapter 8: Area-2

- Area of a circle and sector

Sample Problem:

(Three circles with radius 1 are pairwise tangent to each other. What is the area of the shaded region?



) $\sqrt{3} - \frac{\pi}{2}$

Chapter 9: Analytic Geometry-1

- Cartesian coordinate system (2 dimensions)

- Slope-intercept and point-slope form of a line
- Midpoint and distance formula
- Solving geometry problems by using coordinates

Sample Problem:

(The lines $y = 3x$ and $x = 4$ form a right triangle with the x-axis. Find the slope of a line through the origin that bisects the triangle into two portions of equal area. Express your answer as a common fraction in reduced form.) 3/2

Chapter 10: Analytic Geometry-2

- Reflecting/rotating a point in the coordinate plane
- General equation of a circle in the coordinate plane
- Area of a polygon with Shoelace formula

Sample Problem:

(A vertical line divides the triangle with vertices (0,0), (0,9) and (8,4) into two regions of equal area. Find the equation of the line.) $x = 8 - 4\sqrt{2}$

Chapter 11: 3D-1

- Applications of 3D geometry in the real world
- Applying 2D geometry techniques to 3D space, 3D distance formula
- Surface area of various polyhedra, cylinders, cones, spheres

Sample Problem:

(What is the surface area of a cube inscribed in a sphere with surface area 8π?) 16

Chapter 12: 3D-2

- Volume of various 3D shapes (polyhedra, cylinders, cones, spheres)
- Volume of more complex shapes

Sample Problem:

(An 8×11 sheet of paper is rolled up so that the 11-inch edges align. Find the volume of the resulting cylinder.) $\frac{176}{\pi}$

MC20F Number Theory

Chapter 1: Gauss Sums

- Sums of arithmetic sequences (e.g. sum of the first n positive integers)
- Sum of the first n perfect squares, cubes

Sample Problem:

(The value of the expression

$$1 - 2 - 3 + 4 + 5 - 6 - 7 + 8 + 9 - \ldots + 76 + 77 - 78 - 79$$

is equal to

(A) -98 **(B)** -80 **(C)** -60 **(D)** 40 **(E)** 80) B

Chapter 2: Primes & Prime Factorization

- Definition of divisibility
- Fundamental Theorem of Arithmetic
- Determining if a number is prime or not
- Legendre's formula

Sample Problem:

(The ages of Monica's three children are between 12 and 17. The product of their ages is 3120. What is the sum of their ages?) 44

Chapter 3: Divisibility

- Divisibility rules for all positive integers up to and including 11

Sample Problem:

(What is the smallest integer greater than 200 that is divisible by both 14 and 21?) 210

Chapter 4: Number of Divisors

- Determining the number of divisors of a positive integer n using the prime factorization of n
- Multiplicative functions

Sample Problem:

(The number 144 has A positive divisors, and the number 288 has B positive divisors. What is $B - A$?) 3

Chapter 5: Sum of Divisors

- Definition of $\sigma(n)$
- Determining the sum of divisors of a number n using the prime factorization of n

Sample Problem:

(A positive integer is classified as deficient, perfect, or abundant depending on whether its sum of divisors is less than, equal to, or greater than twice the number itself, respectively. What type of number is 496?) perfect

Chapter 6: Factoring Techniques

- Difference of squares
- Simon's Favorite Factoring Trick (SFFT)
- Sum of cubes, difference of cubes
- Sophie-Germain identity

Sample Problem:

(How many distinct prime factors does $17^4 - 4^4$ have?) 5

Chapter 7: Number Bases

- Representing numbers in different bases
- Converting numbers between bases (emphasis on base 2, 8, and 16)
- Arithmetic in different bases

Sample Problem:

(How many positive integers have the same number of digits in base 5 and base 9?) 64

Chapter 8: GCD & LCM

- Computing the GCD and LCM of two or more numbers
- Euclidean algorithm
- Relation between gcd and lcm ($lcm(a,b) = ab/gcd(a,b)$)

Sample Problem:

(What is $\gcd(637, 2613)$?) 13

Chapter 9: Modular Arithmetic

- Introduction to the congruence operator ($a \equiv b \pmod{m}$)
- Basic properties of modulo (reflexive, symmetric, transitive)
- Computing remainders by finding patterns
- Proof of the divisibility rules for 3, 9, and 11

Sample Problem:

(What are the last two digits of 7^{2023}, when written in base 10?) 43

Chapter 10: FLT and Euler's Totient Theorem

- Applying Fermat's little theorem to find the remainder when a power is divided by a prime

Sample Problem:

(What is the remainder when 19^{19} is divided by 17?) 8

Chapter 11: Chinese Remainder Theorem

- Applying the Chinese remainder theorem to basic modular arithmetic problems
- Solving basic systems of congruences

Sample Problem:

(If a large batch of donuts is arranged into boxes of 10, eight are left over. If arranged into boxes of 12, ten are left over. If arranged into boxes of 14, twelve are left over. Given that there are fewer than 500 donuts, how many donuts are in the batch?) 418

Chapter 12: Diophantine Equations

- Solving linear Diophantine equations of the form $ax + by = c$
- Chicken McNugget theorem
- Bézout's identity
- Pythagorean triples
- Using modular arithmetic to show that a Diophantine equation has no solutions

Sample Problem:

(How many solutions (x, y) in the positive integers are there to $3x + 7y = 1337$?) 63