Topics & Sample Problems

MC20F (AMC 8/MathCounts Basic Fundamentals)

Contents

Part-I

MC20F Algebra

Chapter 1: Integers & Arithmetic

- Order of operations with Integers (PEMDAS)
- Introducing Variables
- Word problems using arithmetic with integers

Sample Problem:

(Cindy was asked by her teacher to subtract 3 from a certain number and then divide the result by 9. Instead, she subtracted 9 and then divided the result by 3, giving an answer of 43. What would her answer have been had she worked the problem correctly?

(A) 15 **(B)** 34 **(C)** 43 **(D)** 51 **(E)** 138) A

Chapter 2: Fractions & Decimals

- Different types of fractions (proper/improper fractions, mixed numbers, simplest form)
- Decimals with repeating/terminating digits
- Converting between fractions and decimals
- Adding, subtracting, multiplying, dividing fractions/decimals
- Telescoping sums and products
- Word problems with fractions and decimals

Sample Problem:

(A collection of coins was shared. Mary received 1/3 of the coins, Amir received 1/5 of the coins, and Samita received 1/6 of the coins. The remaining 36 coins were given to Troy. How many coins were in the entire collection?

(A) 84 **(B)** 90 **(C)** 108 **(D)** 120 **(E)** 144) D

Chapter 3: Percent

- Conversions between percent and fractions/decimals
- Word problems involving percent (tax, tip, interest, etc.)
- Compound Interest
- Word problems with percent

Sample Problem:

(Karl had his salary reduced by 10%. He was later promoted and his salary was increased by 10%. If his original salary was $20,000, what is his present salary?

(A) $16,200 **(B)** $19,800 **(C)** $20,000 **(D)** $20,500 **(E)** $24,000) B

Chapter 4: Exponents

- Basic properties of exponents (multiplying, dividing, raising an exponent to another exponent)
- Negative exponents
- Word problems with exponents

Sample Problem:

(According to the standard convention for exponentiation,

$$2^{2^{2^2}} = 2^{(2^{(2^2)})} = 2^{16} = 65,536.$$

If the order in which the exponentiations are performed is changed, how many other values are possible?

(A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** 4) B

Chapter 5: Radicals

- Square roots, cube roots, simplest radical form
- Negative/fractional exponents
- Rationalizing the denominator, simplifying radicals

Sample Problem:

(What is the value of $(\sqrt{12}+\sqrt{75}+\sqrt{108})^2$?) 507

Chapter 6: Systems of Equations

- Converting a word problem into mathematical equations
- Solving two-unknown linear equations

Sample Problem:

(Farmer Fred said to Farmer John: "If you sell me 45 hectares of land, I will have twice as much land as you." Then Farmer John said to Farmer Fred: "If you sell me 45 hectares of land, I will have just as much land as you." How many hectares of land does farmer Fred have?

(A) 135 **(B)** 180 **(C)** 225 **(D)** 270 **(E)** 315) E

Chapter 7: Distance, Rate, and Time

- Unit conversions
- Distance $=$ Rate $\times$ Time
- Average speed, relative speed
- Problems involving the amount of work/output done

Sample Problem:

(Richard goes on a 6-mile jog one morning. He jogs the first two miles at an average speed of 6 mph. He progressively slows down; his average speed during the next two miles is 4 mph. He walks the remaining two miles at an average speed of 3 mph. What is Richard's average speed, in miles per hour?) 4

Chapter 8: Statistics

- Mean, median, mode, range
- Weighted average

Sample Problem:

(The average age of the 6 people in Room A is 40. The average age of the 4 people in Room B is 25. If the two groups are combined, what is the average age of all the people?

(A) 32.5 **(B)** 33 **(C)** 33.5 **(D)** 34 **(E)** 35) D

Chapter 9: Sequences and Series

- Arithmetic and geometric sequences
- Geometric series (finite and infinite)
- Recursively defined sequences (e.g. the Fibonacci sequence)

Sample Problem:

(A ball is dropped from a height of 3 meters. On its first bounce it rises to a height of 2 meters. It keeps falling and bouncing to $\frac{2}{3}$ of the height it reached in the previous bounce. On which bounce will it not rise to a height of 0.5 meters?

(A) 3 **(B)** 4 **(C)** 5 **(D)** 6 **(E)** 7) C

Chapter 10: Functions & Operations

- Definitions of function, domain, range
- Linear functions ($f(x) = ax + b$)
- Piecewise-defined functions
- Absolute value, floor/ceiling value
- Operators

Sample Problem:

(Let $a@b = (a-1)(b-1)+1$. Find $((3@4)@5)@6$.) 121

Chapter 11: Polynomials-1

- Polynomials of a single variable
- Definitions of degree, coefficient, root
- Quadratic polynomials and the quadratic formula

Sample Problem:

(Let m and n be roots of the polynomial $x^2 - 28x + 192$. What is the unique quadratic polynomial whose leading coefficient is 1 and whose roots are $-m$ and $-n$?) $x^2 + 28x + 192$

Chapter 12: Polynomials-2

- Sum and product of the roots of a quadratic
- Vieta's formulas for cubic and higher degree polynomials

Sample Problem:

(Let a and g be roots of the polynomial $x^2 - 60x + 899 = 0$. What is $a^2 + g^2$?) 1802

MC20F Counting

Chapter 1: Addition/Multiplication Principles

- Addition (rule of sum)
- Multiplication (rule of product)

Sample Problem:

(How many ways can the numbers 1, 2, 3, 4 and 5 be placed in a line so that neither 1 nor 5 occupy either the first or the last place in the sequence?

(A) 6 **(B)** 24 **(C)** 36 **(D)** 54 **(E)** 72) C

Chapter 2: Permutations

- Factorials, permutations
- Counting the number of permutations of n objects taken k at a time

Sample Problem:

(Bethany, Chun, Dominic, and Emily go to the movies. They choose a row with four consecutive empty seats. If Dominic and Emily must sit beside each other, in how many different ways can the four friends sit?

(A) 6 **(B)** 5 **(C)** 12 **(D)** 30 **(E)** 3) C

Chapter 3: Combinations

- Difference between permutations and combinations
- How to compute combinations ("n choose k")

Sample Problem:

(A math club containing $n \geq 4$ members notices that the number of possible 4-member committees is equal to the number of possible 6-member committees. What is the value of n?) 10

Chapter 4: Casework

- Using casework to solve a variety of counting problems that can't be computed directly
- Use casework to break difficult problems into easier pieces

Sample Problem:

(A committee of five people is selected from seven men and six women. How many ways are there to select the committee so that there are at least two men and two women on the committee?) 945

Chapter 5: Complementary Counting & Overcounting

- Applying the techniques of complementary counting or overcounting to solve problems that would be difficult otherwise

Sample Problem:

(An ATM password at Fred's Bank is composed of four digits from 0 to 9, with repeated digits allowable. If no password may begin with the sequence 9, 1, 1, then how many passwords are possible?

(A) 30 **(B)** 7290 **(C)** 9000 **(D)** 9990 **(E)** 9999) D

Chapter 6: Counting Sets

- Definitions of set, subset, size, union, and intersection
- Principle of Inclusion-Exclusion

Sample Problem:

(There are 25 students in a class. 12 of them are on the football team and 14 are on the soccer team. If 3 students are on neither of these teams, how many students are on both the football and soccer teams?) 4

Chapter 7: Counting Shapes & Paths

- Counting the number of paths in a lattice grid using combinations and permutations
- Counting shapes or paths systematically (e.g. without counting manually)

Sample Problem:

(How many rectangles of any size are in the figure below?

) 60

Chapter 8: Counting with Digits

- Various counting problems involving digits
- Palindromic numbers

Sample Problem:

(How many 7-digit numbers are there such that four of its digits are 0, and the rest of the digits are odd?) 1875

Chapter 9: Stars and Bars

- Applying the stars and bars (or "balls and boxes") technique to solve various counting problems

Sample Problem:

(How many positive integer solutions (x, y, z, w) are there to

$$x + y + z + w = 15?$$

) 364

Chapter 10: Binomial & Pascal's Triangle

- Binomial theorem (expanding $(x + y)^n$)
- Pascal's triangle

Sample Problem:

(Given that $\binom{100}{0} + 2\binom{100}{1} + 4\binom{100}{2} + 8\binom{100}{3} + \ldots + 2^{100}\binom{100}{100} = a^b$, where a and b are positive integers and a is as small as possible, what is $a + b$?) 103

Chapter 11: Probability-1

- Definition, of probability
- Sample space, independent/dependent events, disjoint events

Sample Problem:

(Three fair six-sided dice are rolled. What is the probability that **at least** one 6 is rolled? Express your answer as a common fraction in reduced form.) 91/216

Chapter 12: Probability-2

- Expected value and linearity of expectation
- Conditional probability, Bayes' theorem
- Geometric probability

Sample Problem:

(x and y are two positive real numbers chosen randomly and uniformly in the interval $[0, 2]$. What is the probability that $x^2 + y^2 \geq 1$ and $y \geq x$?) $\frac{16-\pi}{32}$