MC45G
AIME Advanced Geometry

Chapter 1: Angles

- Angles in circles and polygons; cyclic quadrilaterals
- Using angle chasing to solve problems

Sample Problem:
(CHMMC-2016-Individual-9) In quadrilateral $ABCD$, $AB = DB$ and $AD = BC$. If $m\angle ABD = 36°$ and $m\angle BCD = 54°$, find $m\angle ADC$ in degrees.

Chapter 2: Special Triangles

- equilateral, 30-60-90, 45-45-90, 15-75-90, 45-60-75, 36-72-72, and 18-72-90 triangles
- Pythagorean triples and Heronian scalenes

Sample Problem: (Prasolov1 p101 q5.24) Points D and E divide sides AC and AB of an equilateral triangle ABC in the ratio of $AD : DC = BE : EA = 1 : 2$. Lines BD and CE meet at point O. Prove that $\angle AOC = 90°$.

Chapter 3: Similarity

- Similarity and congruence conditions (SSS, ASA, SAA, AA and SAS similarity, not SSA)
- Angle bisector theorem

Sample Problem:
(PUMaC-2016-Geometry-7) Let $ABCD$ be a cyclic quadrilateral with circumcircle ω and let AC and BD intersect at X. Let the line through A parallel to BD intersect line CD at E and ω at $Y \neq A$. If $AB = 10$, $AD = 24$, $XA = 17$, and $XB = 21$, then the area of ΔDEY can be written in simplest form as $\frac{m}{n}$. Find $m + n$.

Chapter 4: Special Points

- Properties of the four triangle centers (centroid, orthocenter, incenter, circumcenter)
- The Euler line

Sample Problem:
(AIME-2010-I-15) In $\triangle ABC$ with $AB = 12$, $BC = 13$, and $AC = 15$, let M be a point on $\overline{AC}$ such that the incircles of $\triangle ABM$ and $\triangle BCM$ have equal radii. Let p and q be positive relatively prime integers such that $\frac{AM}{CM} = \frac{p}{q}$. Find $p + q$.

Chapter 5: Length-1

- Triangle inequality and Ravi substitution
- Pythagorean theorem and distance formula
- Mass points
- Ceva's Theorem, Menelaus' Theorem and Stewart's Theorem

Sample Problem:

Chapter 6: Length-2

- Length problems involving circles
- Power of a point
- Radical axis and radical center
- Ptolemy's theorem

Sample Problem:
(PUMaC-2014-Geometry-4) Consider the cyclic quadrilateral with sides 1, 4, 8, 7 in that order. What is its circumdiameter? Let the answer be of the form $a\sqrt{b}+c$, for b square free. Find $a+b+c$.

Chapter 7: Area-1

- Triangle area formulas
- Special quadrilateral area formulas such as Brahmagupta's formula

Sample Problem:
(AIME-2018-I-12) For each subset T of $U = \{1, 2, 3, \cdots, 18\}$, let $s(T)$ be the sum of the elements of T, with $s(\varnothing)$ defined to be 0. If T is chosen at random among all subsets of U, the probability that $s(T)$ is divisible by 3 is $\frac{m}{n}$, where m and n are relatively prime positive integers. Find m.

Chapter 8: Area-2

- Area problems involving length ratios

Sample Problem:
(HMMT Feb-2004-Geometry-10) Right triangle XYZ has right angle at Y and $XY = 228$, $YZ = 2004$. Angle Y is trisected, and the angle trisectors intersect XZ at P and Q so that X, P, Q, Z lie on XZ in that order. Find the value of $(PY+YZ)(QY+XY)$.

Chapter 9: Trigonometry

- Definitions of trigonometric functions, basic trig identities, sum and difference formulas
- Law of sines, law of cosines, ratio lemma
- Trigonometric Ceva

Sample Problem:
(AIME-2018-I-13) Let ΔABC have side lengths $AB = 30$, $BC = 32$, and $AC = 34$. Point X lies in the interior of $\overline{BC}$, and points I_1 and I_2 are the incenters of ΔABX and ΔACX, respectively. Find the minimum possible area of ΔAI_1I_2 as X varies along BC.

Chapter 10: Analytic Geometry

- Distance formulas (between two points, point & line)
- The slope and the equation of a line (slope-intercept and point slope)
- Reflections over lines
- Equation of circles
- Shoelace formula and Pick's theorem

Sample Problem:
(HMMT Feb-2010-Guts-13) A triangle in the xy-plane is such that when projected onto the x-axis, y-axis, and the line $y = x$, the results are line segments whose endpoints are $(1, 0)$ and $(5, 0)$, $(0, 8)$ and $(0, 13)$, and $(5, 5)$ and $(7.5, 7.5)$, respectively. What is the triangle's area?

Chapter 11: Complex Numbers

- Introduction to radians, Euler's formula
- Various representations of complex numbers
- Magnitude, argument and distance in complex plane
- Rotations, colinearity, perpendicularity

Sample Problem:
(AIME-2012-I-14) Complex numbers a, b, and c are zeros of a polynomial $P(z) = z^3 + qz + r$, and $|a|^2 + |b|^2 + |c|^2 = 250$. The points corresponding to a, b, and c in the complex plane are the vertices of a right triangle with hypotenuse h. Find h^2.

Chapter 12: 3D

- Platonic solids, spheres, cylinders, cones
- Distance formula, point-to-plane formula, Euler characteristic
- Using cross-sections and 2D properties to solve 3D problems

Sample Problem:
(PUMaC-2010-Geometry-5) A cuboctahedron is a solid with 6 square faces and 8 equilateral triangle faces, with each edge adjacent to both a square and a triangle (see picture). Suppose the ratio of the volume of an octahedron to a cuboctahedron with the same side length is r. Find $100r^2$.