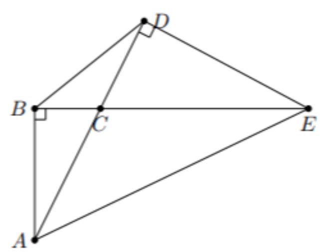
MC40G
AIME Basic Geometry

Chapter 1: Angles

- Angles in circles and polygons; cyclic quadrilaterals
- Using angle chasing to solve problems

Sample Problem:
(CHMMC-2012 Fall-Team-4) Consider the figure below, not drawn to scale.

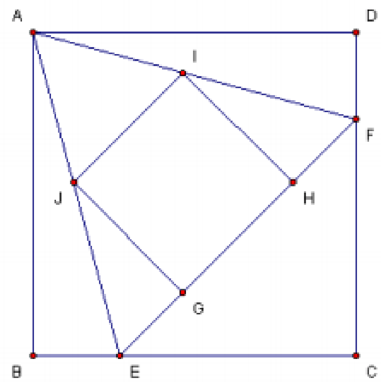


In this figure, assume that $AB \perp BE$ and $AD \perp DE$. Also, let $AB = \sqrt{6}$ and $\angle BED = \frac{\pi}{6}$. Find AC.

Chapter 2: Special Triangles

- equilateral, 30-60-90, 45-45-90, 15-75-90, 45-60-75, 36-72-72, and 18-72-90 triangles
- Pythagorean triples and Heronian scalenes

Sample Problem: (SMT-2011-Team-1) Let $ABCD$ be a unit square. The point E lies on BC and F lies on AD. $\triangle AEF$ is equilateral. $GHIJ$ is a square inscribed in $\triangle AEF$ so that GH is on EF. Compute the area of $GHIJ$.



Chapter 3: Similarity

- Similarity and congruence conditions (SSS, ASA, SAA, AA and SAS similarity, not SSA)
- Angle bisector theorem

Sample Problem:
(AMC12-2002-A23) In triangle ABC, side $\overline{AC}$ and the perpendicular bisector of $\overline{BC}$ meet in point D, and $\overline{BD}$ bisects $\angle ABC$. If $AD = 9$ and $DC = 7$, what is the area of triangle ABD?

(A) 14 **(B)** 21 **(C)** 28 **(D)** $14\sqrt{5}$ **(E)** $28\sqrt{5}$

Chapter 4: Special Points

- Properties of the four triangle centers (centroid, orthocenter, incenter, circumcenter)
- The Euler line

Sample Problem:
(AIME-2016-I-6) In $\triangle ABC$ let I be the center of the inscribed circle, and let the bisector of $\angle ACB$ intersect AB at L. The line through C and L intersects the

circumscribed circle of $\triangle ABC$ at the two points C and D. If $LI = 2$ and $LD = 3$, then $IC = \frac{p}{q}$, where p and q are relatively prime positive integers. Find $p + q$.

Chapter 5: Length-1

- Triangle inequality and Ravi substitution
- Pythagorean theorem and distance formula
- Mass points
- Ceva's Theorem, Menelaus' Theorem and Stewart's Theorem

Sample Problem:
(AIME-2008-II-5) In trapezoid $ABCD$ with $\overline{BC} \parallel \overline{AD}$, let $BC = 1000$ and $AD = 2008$. Let $\angle A = 37°$, $\angle D = 53°$, and M and N be the midpoints of $\overline{BC}$ and $\overline{AD}$, respectively. Find the length MN.

Chapter 6: Length-2

- Length problems involving circles
- Power of a point
- Radical axis and radical center
- Ptolemy's theorem

Sample Problem:
(PUMaC-2010-Geometry-6) A semicircle is folded along a chord AN and intersects its diameter MN at B. Given that $MB : BN = 2 : 3$ and $MN = 10$, if $AN = x$, find x^2.

Chapter 7: Area-1

- Triangle area formulas
- Special quadrilateral area formulas such as Brahmagupta's formula

Sample Problem:
(AIME-2016-II-7) Squares $ABCD$ and $EFGH$ have a common center and $\overline{AB}||\overline{EF}$. The area of $ABCD$ is 2016, and the area of $EFGH$ is a smaller positive integer. Square $IJKL$ is constructed so that each of its vertices lies on a side of $ABCD$ and each vertex of $EFGH$ lies on a side of $IJKL$. Find the difference between the largest and smallest positive integer values for the area of $IJKL$.

Chapter 8: Area-2

- Area problems involving length ratios

Sample Problem:
(BMT-2014-Individual-12) Suppose four coplanar points A, B, C, and D satisfy $\overline{AB} = 3, \overline{BC} = 4, \overline{CA} = 5$, and $\overline{BD} = 6$. Determine the maximal possible area of ΔACD.

Chapter 9: Trigonometry

- Definitions of trigonometric functions, basic trig identities, sum and difference formulas
- Law of sines, law of cosines, ratio lemma
- Trigonometric Ceva

Sample Problem:
(AMC12-2018-A23) In $\triangle PAT, \angle P = 36^\circ, \angle A = 56^\circ$, and $PA = 10$. Points U and G lie on sides $\overline{TP}$ and $\overline{TA}$, respectively, so that $PU = AG = 1$. Let M and N be the midpoints of segments $\overline{PA}$ and $\overline{UG}$, respectively. What is the degree measure of the acute angle formed by lines MN and PA?

(A) 76 **(B)** 77 **(C)** 78 **(D)** 79 **(E)** 80

Chapter 10: Analytic Geometry

- Distance formulas (between two points, point & line)
- The slope and the equation of a line (slope-intercept and point slope)

- Reflections over lines
- Equation of circles
- Shoelace formula and Pick's theorem

Sample Problem:
(AMC12-2014-A25) The parabola P has focus $(0,0)$ and goes through the points $(4,3)$ and $(-4,-3)$. For how many points $(x,y) \in P$ with integer coordinates is it true that $|4x+3y| \leq 1000$?

(A) 38 **(B)** 40 **(C)** 42 **(D)** 44 **(E)** 46

Chapter 11: Complex Numbers

- Introduction to radians, Euler's formula
- Various representations of complex numbers
- Magnitude, argument and distance in complex plane
- Rotations, colinearity, perpendicularity

Sample Problem:
(Math Day at the Beach-2010-Team-6) Let $z_1,\ z_2,\ \ldots,\ z_{10}$ be complex numbers that form a regular decagon (10-sided polygon) in the complex plane, with that decagon inscribed in a circle of radius $\sqrt[5]{7}$ centered at 2. At least one of the z_k is real. Compute the product $z_1 z_2 \cdots z_{10}$.

Chapter 12: 3D

- Platonic solids, spheres, cylinders, cones
- Distance formula, point-to-plane formula, Euler characteristic
- Using cross-sections and 2D properties to solve 3D problems

Sample Problem:
(AIME-2000-I-8) A container in the shape of a right circular cone is 12 inches tall and its base has a 5-inch radius. The liquid that is sealed inside is 9 inches deep

when the cone is held with its point down and its base horizontal. When the cone is held with its point up and its base horizontal, the height of the liquid is $m - n\sqrt[3]{p}$, where m, n, and p are positive integers and p is not divisible by the cube of any prime number. Find $m + n + p$.