MC35C
AMC 10/12 Advanced Counting

Chapter 1: Counting Basics

- Addition/multiplication principles
- Permutations, combinations, binomial coefficients

Sample Problem:
(SMT-2018-General-15) How many ways are there to select distinct integers x, y, where $1 \leq x \leq 25$ and $1 \leq y \leq 25$, such that $x + y$ is divisible by 5?

Chapter 2: Casework

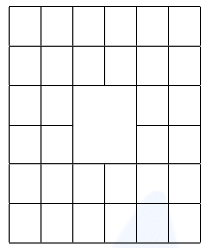
- Solving a variety of counting problems using casework
- Use casework to break difficult problems into easier pieces

Sample Problem:
(HMMT Feb-2006-Combinatorics-6) For how many ordered triplets (a, b, c) of positive integers less than 10 is the product $a \times b \times c$ divisible by 20?

Chapter 3: Complementary Counting & Overcounting

- Solving counting problems using the techniques of complementary counting and/or overcounting

Sample Problem:
(HMMT Feb-2008-Guts-6) Determine the number of non-degenerate rectangles whose edges lie completely on the grid lines of the following figure.



Chapter 4: Counting Sets

- Definitions of set, cardinality, union, intersection
- Principle of inclusion-exclusion for two or more sets

Sample Problem:
(HMMT Feb-2010-Combinatorics-1) Let $S = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$. How many (potentially empty) subsets T of S are there such that, for all x, if x is in T and $2x$ is in S then $2x$ is also in T?

Chapter 5: Counting with Digits

- Solving a variety of counting problems involving digits of a number
- Counting palindromes

Sample Problem:
(AMC12-2008-A21) A permutation $(a_1, a_2, a_3, a_4, a_5)$ of $(1, 2, 3, 4, 5)$ is heavy-tailed if $a_1 + a_2 < a_4 + a_5$. What is the number of heavy-tailed permutations?

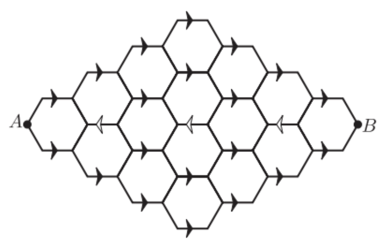
(A) 36 **(B)** 40 **(C)** 44 **(D)** 48 **(E)** 52

Chapter 6: Path Counting & Bijections

- Definitions of injective, surjective, and bijective functions

- Examples of bijections between two infinite sets (e.g. the set of whole numbers and the set of integers)
- Solving counting problems by establishing a bijection

Sample Problem:
(AMC10-2012-B25) A bug travels from A to B along the segments in the hexagonal lattice pictured below. The segments marked with an arrow can be traveled only in the direction of the arrow, and the bug never travels the same segment more than once. How many different paths are there?



(A) 2112 **(B)** 2304 **(C)** 2368 **(D)** 2384 **(E)** 2400

Chapter 7: Stars and Bars

- Using the stars and bars technique to solve a variety of counting problems

Sample Problem:
(Caleb Ji) How many ways can David pick four of the first twelve positive integers such that no two of the numbers he picks are consecutive?

Chapter 8: Binomial

- Binomial theorem, Pascal's triangle, Sierpinski's triangle
- Various combinatorial identities, such as the hockey stick identity

Sample Problem:
(AMC10-2011-B23) What is the hundreds digit of 2011^{2011}?

(A) 1 **(B)** 4 **(C)** 5 **(D)** 6 **(E)** 9

Chapter 9: Counting with Recursion

- Solving counting problems by setting up a recursion and/or finding patterns

Sample Problem:
(Lehigh MC-2014-26) How many 10-digit strings of 0's and 1's are there that do not contain any consecutive 0's?

Chapter 10: Probability-1

- Basic probability definitions and axioms
- Definitions of complementary events, independence, disjoint events

Sample Problem:
(AMC10-2004-B23) Each face of a cube is painted either red or blue, each with probability $\dfrac{1}{2}$. The color of each face is determined independently. What is the probability that the painted cube can be placed on a horizontal surface so that the four vertical faces are all the same color?

(A) $\dfrac{1}{4}$ **(B)** $\dfrac{5}{16}$ **(C)** $\dfrac{3}{8}$ **(D)** $\dfrac{7}{16}$ **(E)** $\dfrac{1}{2}$

Chapter 11: Probability-2

- Conditional probability, Bayes' theorem
- Geometric probability

Sample Problem:
(AMC10-2012-A25) Real numbers x, y, and z are chosen independently and at random from the interval $[0, n]$ for some positive integer n. The probability that no two of x, y, and z are within 1 unit of each other is greater than $\frac{1}{2}$. What is the smallest possible value of n?

(A) 7 **(B)** 8 **(C)** 9 **(D)** 10 **(E)** 11

Chapter 12: Expected Value

- Expected value and linearity of expectation (for an arbitrary number of events)
- Introduction to state diagrams, Markov chains

Sample Problem:
(HMMT Nov-2010-General1-4) An ant starts at the point $(1, 0)$. Each minute, it walks from its current position to one of the four adjacent lattice points until it reaches a point (x, y) with $|x| + |y| \geq 2$. What is the probability that the ant ends at the point $(1, 1)$?