MC35A
AMC 10/12 Advanced Algebra

Chapter 1: Arithmetic

- Word problems using arithmetic with integers, fractions, decimals, and percent
- Decimals with repeating/terminating digits
- Rational/Irrational numbers

Sample Problem:
(AMC12-2002-A20) Suppose that a and b are digits, not both nine and not both zero, and the repeating decimal $0.\overline{ab}$ is expressed as a fraction in lowest terms. How many different denominators are possible?

(A) 3 **(B)** 4 **(C)** 5 **(D)** 8 **(E)** 9

Chapter 2: Exponents & Radicals

- Properties of exponents and radicals
- Negative/fractional exponents
- Rationalizing the denominator, simplifying radicals
- Using conjugates of radicals

Sample Problem:
(Lehigh MC-2016-34) What is the smallest integer larger than $(\sqrt{5}+\sqrt{3})^6$

Chapter 3: Word Problems & System of Equations

- Word problems, systems of equations in two or more variables

Sample Problem:
(AMC10-2002-B20) Let a, b, and c be real numbers such that $a - 7b + 8c = 4$ and $8a + 4b - c = 7$. Then $a^2 - b^2 + c^2$ is

(A) 0 **(B)** 1 **(C)** 4 **(D)** 7 **(E)** 8

Chapter 4: Time, Travel, Work

- Distance = Rate $\times$ Time, average speed
- Harmonic mean
- Relative speed
- Rate/Work Problems

Sample Problem:
(AMC10-2012-A19) Paula the painter and her two helpers each paint at constant, but different, rates. They always start at 8:00 AM, and all three always take the same amount of time to eat lunch. On Monday the three of them painted 50% of a house, quitting at 4:00 PM. On Tuesday, when Paula wasn't there, the two helpers painted only 24% of the house and quit at 2:12 PM. On Wednesday Paula worked by herself and finished the house by working until 7:12 P.M. How long, in minutes, was each day's lunch break?

(A) 30 **(B)** 36 **(C)** 42 **(D)** 48 **(E)** 60

Chapter 5: Sequences-1

- Mean, median, mode, range
- Arithmetic and geometric sequences
- Geometric series formula and derivation

Sample Problem:
(AMC10-2000-A23) When the mean, median, and mode of the list $10, 2, 5, 2, 4, 2, x$ are arranged in increasing order, they form a non-constant arithmetic progression. What is the sum of all possible real values of x?

(A) 3 **(B)** 6 **(C)** 9 **(D)** 17 **(E)** 20

Chapter 6: Sequences-2

- Recurrent sequences
- Finding the general term via patterns

Sample Problem:
(Lehigh MC-2008-33) A Fibonacci-like sequence of numbers is defined by $a_1 = 1$, $a_2 = 3$, and for $n \geq 3$, $a_n = a_{n-1} + a_{n-2}$. One can compute that $a_{29} = 1149851$ and $a_{30} = 1860498$. What is the value of $\sum_{n=1}^{28} a_n$?

Chapter 7: Functions & Operations

- Definitions of function, domain, codomain/range
- Injective, surjective, bijective functions
- Inverse functions
- Operators
- Simple functional equations

Sample Problem:
(Lehigh MC-2002-36) If $2f(x) + f(1 - x) = x^2$ for all x, then $f(x) =$

Chapter 8: Polynomials-1

- Polynomials of a single variable; definitions of degree, root, etc.
- Solving for the roots of a quadratic by factoring, completing the square, or quadratic formula

- Rational root theorem
- Fundamental theorem of algebra
- Less emphasis on complex numbers (Chapter 12)

Sample Problem:
(Aaron Lin, David Zhu) Suppose P is a monic quartic polynomial (i.e. a 4th-degree polynomial with leading coefficient 1) such that $P(1) = 1$, $P(2) = 4$, $P(3) = 9$, $P(4) = 16$. Find $P(5)$.

Chapter 9: Polynomials-2

- Generalized Vieta's formulas
- Manipulation of symmetric sums to produce other expressions

Sample Problem:
(HMMT Nov-2016-Guts-27) Let r_1, r_2, r_3, r_4 be the four roots of the polynomial $x^4 - 4x^3 + 8x^2 - 7x + 3$. Find the value of

$$\frac{r_1^2}{r_2^2 + r_3^2 + r_4^2} + \frac{r_2^2}{r_1^2 + r_3^2 + r_4^2} + \frac{r_3^2}{r_1^2 + r_2^2 + r_4^2} + \frac{r_4^2}{r_1^2 + r_2^2 + r_3^2}.$$

Chapter 10: Trigonometry

- Review of trigonometric functions (sin, cos, tan, csc, sec, cot)
- More emphasis on trigonometric identities (addition and multiple-angle formulae)
- Solving algebra problems via trig substitution

Sample Problem:
(BMT-2016-Analysis-5) Find

$$\frac{\tan 1^\circ}{1 + \tan 1^\circ} + \frac{\tan 2^\circ}{1 + \tan 2^\circ} + \cdots + \frac{\tan 89^\circ}{1 + \tan 89^\circ}.$$

Chapter 11: Logarithm

- Definition of a logarithm in base b, simple logarithmic identities (change-of-base formula, addition/subtraction of logarithms)
- Natural logarithms, the number e
- Applications: Binary search, merge sort example

Sample Problem:
(ARML-2010-Team-2) Define $\log^*(n)$ to be the smallest number of times the log function must be iteratively applied to n to get a result less than or equal to 1. For example, $\log^*(1000) = 2$ since $\log 1000 = 3$ and $\log(\log 1000) = \log 3 = 0.477\ldots \leq 1$. Let a be the smallest integer such that $\log^*(a) = 3$. Compute the number of zeros in the base 10 representation of a.

Chapter 12: Complex Numbers

- More rigorous introduction to complex numbers
- Review of the Fundamental Theorem of Algebra, conjugate root theorem
- Polar form, Euler's formula, de Moivre's formula
- Roots of unity

Sample Problem:
(Victor Chen) If $z = -\frac{1}{2} + \frac{\sqrt{3}}{2}i$, compute $z^{100} + \frac{1}{z^{100}}$.