MC30G
AMC 10/12 Basic Geometry

Chapter 1: Angles

- Angles (review)
- Inscribed angles in a circle, cyclic quadrilaterals

Sample Problem:
(Alec Sun) In a regular 9-gon $ABCDEFGHI$, draw a circle that is tangent to IA at A and CD at C. What is the degree measure of minor arc AC?

Chapter 2: Special Triangles

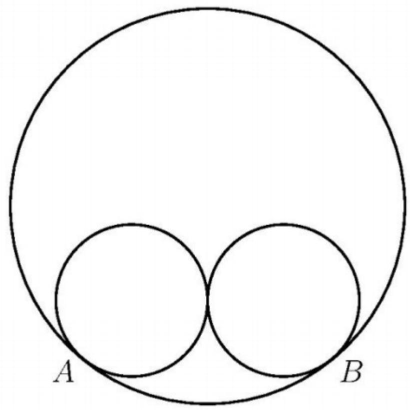
- 30-60-90, 45-45-90, and 15-75-90 triangles
- Pythagorean triples

Sample Problem:
(Math Day at the Beach-2012-Team-1) Each of two congruent equilateral triangles with side s has center that is a vertex of the other triangle. What is the area of the overlap, in terms of s?

Chapter 3: Similarity

- Similarity/congruence axioms (SSS, SAS, ASA, AA similarity)
- Power of a point, angle bisector theorem

Sample Problem:
(AMC10-2018-A15) Two circles of radius 5 are externally tangent to each other and are internally tangent to a circle of radius 13 at points A and B, as shown in the diagram. The distance AB can be written in the form $\frac{m}{n}$, where m and n are relatively prime positive integers. What is $m+n$?

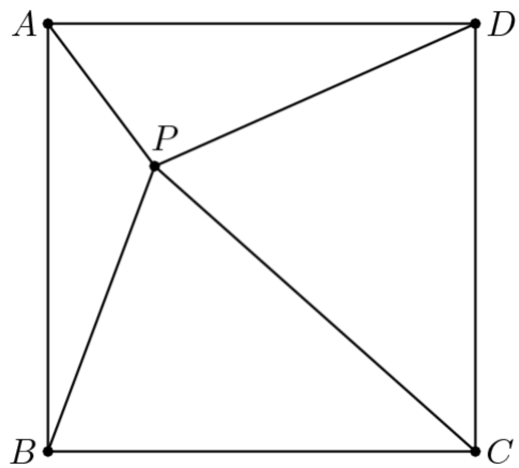


(A) 21 **(B)** 29 **(C)** 58 **(D)** 69 **(E)** 93

Chapter 4: Special Points

- Special points of a triangle (centroid, incenter, circumcenter, orthocenter)

Sample Problem:
(AMC12-2018-B13) Square $ABCD$ has side length 30. Point P lies inside the square so that $AP = 12$ and $BP = 26$. The centroids of $\triangle ABP, \triangle BCP, \triangle CDP$, and $\triangle DAP$ are the vertices of a convex quadrilateral. What is the area of that quadrilateral?



(A) $100\sqrt{2}$ **(B)** $100\sqrt{3}$ **(C)** 200 **(D)** $200\sqrt{2}$ **(E)** $200\sqrt{3}$

Chapter 5: Length-1

- Triangle inequality, Ravi substitution
- Pythagorean theorem, distance formula
- Stewart's theorem

Sample Problem:
(AMC12-2012-A12) A square region $ABCD$ is externally tangent to the circle with equation $x^2 + y^2 = 1$ at the point $(0, 1)$ on the side CD. Vertices A and B are on the circle with equation $x^2 + y^2 = 4$. What is the side length of the square?

(A) $\frac{\sqrt{10}+5}{10}$ **(B)** $\frac{2\sqrt{5}}{5}$ **(C)** $\frac{2\sqrt{2}}{3}$ **(D)** $\frac{2\sqrt{19}-4}{5}$ **(E)** $\frac{9-\sqrt{17}}{5}$

Chapter 6: Length-2

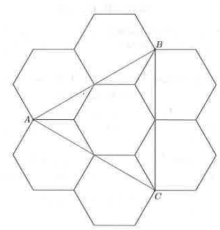
- Mass points considering levers/torque
- Ceva's theorem, Menelaus' theorem

Sample Problem:
(Lehigh MC-2014-27) Let BE be a median of triangle ABC, and let D be a point on AB such that $BD/DA = 3/7$. What is the ratio of the area of triangle BED to that of triangle ABC?

Chapter 7: Area-1

- Areas of simple shapes (triangle, certain quadrilaterals)
- Triangle area formulas (Heron's formula, A = rs, A = abc/4R, (ab sin C)/2

Sample Problem:
(AMC10-2014-B13) Six regular hexagons surround a regular hexagon of side length 1 as shown. What is the area of $\triangle ABC$?



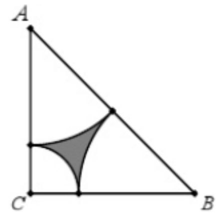
(A) $2\sqrt{3}$ **(B)** $3\sqrt{3}$ **(C)** $1+3\sqrt{2}$ **(D)** $2+2\sqrt{3}$ **(E)** $3+2\sqrt{3}$

Chapter 8: Area-2

- Area formula for a circle, sector
- Brahmagupta's formula
- Area of more complicated shapes involving circles and/or other polygons

Sample Problem:
(ARML-0000-Team-1) In ΔABC, $m\angle A = m\angle B = 45°$ and $AB = 16$. Mutually tangent circular arcs are drawn centered at all three vertices; the arcs centered at A and B intersect at the midpoint of $\overline{AB}$. Compute the area of the region inside the

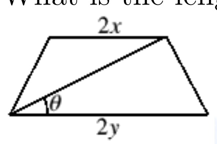
triangle and outside of the three arcs.



Chapter 9: Trigonometry-1

- Definitions of sin, cos, tan, as well as csc, sec, cot
- The unit circle
- Basic trig identities, Sum and difference formulas for sin, cos, tan (e.g. sin(a+b))

Sample Problem:
(UK MC-2010-Senior-14) The parallel sides of a trapezium have lengths $2x$ and $2y$ respectively. The diagonals are equal in length, and a diagonal makes an angle θ with the parallel sides, as shown. What is the length of each diagonal?



(A) $x+y$ **(B)** $\dfrac{x+y}{\sin\theta}$ **(C)** $(x+y)\cos\theta$ **(D)** $(x+y)\tan\theta$ **(E)** $\dfrac{x+y}{\cos\theta}$

Chapter 10: Trigonometry-2

- Law of sines
- Law of cosines
- Ratio lemma, trig Ceva's theorem
- Solving algebra problems by trig substitution

Sample Problem:
(Hong Kong MC-2006-17) If a square can completely cover a triangle with side lengths 3, 4 and 5, find the smallest possible side length of the square.

Chapter 11: Analytic Geometry

- Slope, equation of a line using slope-intercept or point-slope form, distance and midpoint formulas
- Reflections over lines in the coordinate plan
- Equation of a circle
- Shoelace formula

Sample Problem:
(AMC12-2006-B16) Regular hexagon $ABCDEF$ has vertices A and C at $(0, 0)$ and $(7, 1)$, respectively. What is its area?

(A) $20\sqrt{3}$ **(B)** $22\sqrt{3}$ **(C)** $25\sqrt{3}$ **(D)** $27\sqrt{3}$ **(E)** 50

Chapter 12: 3D

- Distance formula in 3D
- Area/volume of various 3D shapes (cube, prisms, cylinders, cones, spheres)
- Common 3D solids
- Euler's polyhedral formula

Sample Problem:
(HMMT Feb-2004-Guts-18) On a spherical planet with diameter 10,000 km, powerful explosives are placed at the north and south poles. The explosives are designed to vaporize all matter within 5,000 km of ground zero and leave anything beyond 5,000 km untouched. After the explosives are set off, what is the new surface area of the planet, in square kilometers?