MC30C
AMC 10/12 Basic Counting

Chapter 1: Counting Basics

- Addition/multiplication principles
- Permutations, combinations, binomial coefficients

Sample Problem:
(Justin Stevens) How many subsets does the set of odd numbers $\{1, 3, 5, 7, 9, \cdots, 19\}$ have?

Chapter 2: Casework

- Solving a variety of counting problems using casework
- Use casework to break difficult problems into easier pieces

Sample Problem:
(AMC12-2014-A13) A fancy bed and breakfast inn has 5 rooms, each with a distinctive color-coded decor. One day 5 friends arrive to spend the night. There are no other guests that night. The friends can room in any combination they wish, but with no more than 2 friends per room. In how many ways can the innkeeper assign the guests to the rooms?

(A) 2100 **(B)** 2220 **(C)** 3000 **(D)** 3120 **(E)** 3125

Chapter 3: Complementary Counting & Overcounting

- Solving counting problems using the techniques of complementary counting and/or overcounting

Sample Problem:

Chapter 4: Counting Sets

- Definitions of set, cardinality, union, intersection
- Principle of inclusion-exclusion for two or more sets

Sample Problem:
(Justin Stevens) There are 140 students in my high school. 70 of them play basketball, 100 of them play soccer, and 30 play hockey. 44 play both soccer and basketball, 12 play basketball and hockey, and 9 play soccer and hockey. How many students play all three sports?

Chapter 5: Counting with Digits

- Solving a variety of counting problems involving digits of a number
- Counting palindromes

Sample Problem:
(Jamie Gu) How many 5-digit positive integers have exactly three 5's?

Chapter 6: Path Counting & Bijections

- Definitions of injective, surjective, and bijective functions
- Examples of bijections between two infinite sets (e.g. the set of whole numbers and the set of integers)
- Solving counting problems by establishing a bijection

Sample Problem:
(Evan Chen) Determine the number of sequences of positive integers $1 = x_0 < x_1 < \dots < x_{10} = 10^5$ with the property that for each $m = 0, \dots, 9$ the number $\frac{x_{m+1}}{x_m}$ is a prime number.

Chapter 7: Stars and Bars

- Using the stars and bars technique to solve a variety of counting problems

Sample Problem:

Chapter 8: Binomial

- Binomial theorem, Pascal's triangle, Sierpinski's triangle
- Various combinatorial identities, such as the hockey stick identity

Sample Problem:
(AlphaStar) What is the coefficient of x^{42} in the expansion of $(1 + x + x^2 + \cdots + x^{39})(1 + x + \cdots + x^{40})(1 + x + \cdots + x^{41})$?

Chapter 9: Counting with Recursion

- Solving counting problems by setting up a recursion and/or finding patterns

Sample Problem:
(Ali Gurel) Gustavo is filling his 2×10 room completely with 10 carpets of size 1×2. In how many different ways can he do this?

Chapter 10: Probability-1

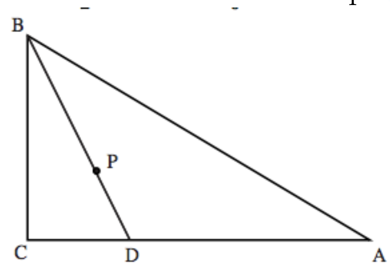
- Basic probability definitions and axioms
- Definitions of complementary events, independence, disjoint events

Sample Problem: (AIME-2002-I-1) Many states use a sequence of three letters followed by a sequence of three digits as their standard license-plate pattern. Given that each three-letter three-digit arrangement is equally likely, the probability that such a license plate will contain at least one palindrome (a three-letter arrangement or a three-digit arrangement that reads the same left-to-right as it does right-to-left) is m/n, where m and n are relatively prime positive integers. Find $m+n$.

Chapter 11: Probability-2

- Conditional probability, Bayes' theorem
- Geometric probability

Sample Problem:
(AMC12-2002-A22) Triangle ABC is a right triangle with $\angle ACB$ as its right angle, $m\angle ABC = 60°$, and $AB = 10$. Let P be randomly chosen inside $\triangle ABC$, and extend $\overline{BP}$ to meet $\overline{AC}$ at D. What is the probability that $BD > 5\sqrt{2}$?



(A) $\frac{2-\sqrt{2}}{2}$ **(B)** $\frac{1}{3}$ **(C)** $\frac{3-\sqrt{3}}{3}$ **(D)** $\frac{1}{2}$ **(E)** $\frac{5-\sqrt{5}}{5}$

Chapter 12: Expected Value

- Expected value and linearity of expectation (for an arbitrary number of events)
- Introduction to state diagrams, Markov chains

Sample Problem:
(AMC12-2016-B19) Tom, Dick, and Harry are playing a game. Starting at the same time, each of them flips a fair coin repeatedly until he gets his first head, at which

point he stops. What is the probability that all three flip their coins the same number of times?

(A) $\frac{1}{8}$ **(B)** $\frac{1}{7}$ **(C)** $\frac{1}{6}$ **(D)** $\frac{1}{4}$ **(E)** $\frac{1}{3}$