MC30A
AMC 10/12 Basic Algebra

Chapter 1: Arithmetic

- Word problems using arithmetic with integers, fractions, decimals, and percent
- Decimals with repeating/terminating digits
- Rational/Irrational numbers

Sample Problem: (AMC10-2009-A5) What is the sum of the digits of the square of $111,111,111$?

(A) 18 **(B)** 27 **(C)** 45 **(D)** 63 **(E)** 81

Chapter 2: Exponents & Radicals

- Properties of exponents and radicals
- Negative/fractional exponents
- Rationalizing the denominator, simplifying radicals
- Using conjugates of radicals

Sample Problem:
(Lehigh MC-2008-10) Simplify $\sqrt{19+\sqrt{297}}-\sqrt{19-\sqrt{297}}$.

Chapter 3: Word Problems & System of Equations

- Word problems, systems of equations in two or more variables

Sample Problem:
(SMT-2012-General-4) Steve works 40 hours a week at his new job. He usually gets paid 8 dollars an hour, but if he works for more than 8 hours on a given day, he earns 12 dollars an hour for every additional hour over 8 hours. If x is the maximum number of dollars that Steve can earn in one week by working exactly 40 hours, and y is the minimum number of dollars that Steve can earn in one week by working exactly 40 hours, what is $x - y$?

Chapter 4: Time, Travel, Work

- Distance = Rate $\times$ Time, average speed
- Harmonic mean
- Relative speed
- Rate/Work Problems

Sample Problem:
(AMC10-2002-A12) Mr. Earl E. Bird leaves his house for work at exactly 8:00 A.M. every morning. When he averages 40 miles per hour, he arrives at his workplace three minutes late. When he averages 60 miles per hour, he arrives three minutes early. At what average speed, in miles per hour, should Mr. Bird drive to arrive at his workplace precisely on time?

(A) 45 **(B)** 48 **(C)** 50 **(D)** 55 **(E)** 58

Chapter 5: Sequences-1

- Mean, median, mode, range
- Arithmetic and geometric sequences
- Geometric series formula and derivation

Sample Problem:
(AMC10-2010-B17) Every high school in the city of Euclid sent a team of 3 students to a math contest. Each participant in the contest received a different score. Andrea's score was the median among all students, and hers was the highest score on her team. Andrea's teammates Beth and Carla placed 37^{th} and 64^{th}, respectively. How many schools are in the city?

(A) 22 **(B)** 23 **(C)** 24 **(D)** 25 **(E)** 26

Chapter 6: Sequences-2

- Recurrent sequences
- Finding the general term via patterns

Sample Problem:
(Math Day at the Beach-2018-Individual-14) Form the sequence such that $x_1 = x_2 = 1$, and for $n > 2$, $x_n = x_{n-1}^2 + x_{n-2}$. Of the numbers $x_1,\ x_2, \ldots, x_{2018}$, how many are divisible by 3?

Chapter 7: Functions & Operations

- Definitions of function, domain, codomain/range
- Injective, surjective, bijective functions
- Inverse functions
- Operators
- Simple functional equations

Sample Problem:

(Math Day at the Beach-2014-Individual-18) Compute $\sum_{n=1}^{99} \lfloor 0.67n \rfloor$,where the notation $\lfloor x \rfloor$ means the greatest integer that is less than or equal to x.

Chapter 8: Polynomials-1

- Polynomials of a single variable; definitions of degree, root, etc.
- Solving for the roots of a quadratic by factoring, completing the square, or quadratic formula
- Rational root theorem
- Fundamental theorem of algebra
- Less emphasis on complex numbers (Chapter 12)

Sample Problem:
(HMMT Nov-2012-Guts-15) Find the area of the region in the xy-plane consisting of all points (a, b) such that the quadratic $ax^2 + 2(a + b - 7)x + 2b = 0$ has fewer than two real solutions for x.

Chapter 9: Polynomials-2

- Generalized Vieta's formulas
- Manipulation of symmetric sums to produce other expressions

Sample Problem:
(Justin Stevens) Find $(3 - r)(3 - s)(3 - t)$ if r, s, and t are the roots of $f(x) = 3x^3 - 9x^2 + 3x - 7$. Express your answer as a common fraction in reduced form.

Chapter 10: Trigonometry

- Review of trigonometric functions (sin, cos, tan, csc, sec, cot)
- More emphasis on trigonometric identities (addition and multiple-angle formulae)
- Solving algebra problems via trig substitution

Sample Problem:
(Richard Spence) How many solutions $\theta \in [0, 2\pi)$ are there such that $\sin\theta = \sin 6\theta$?

Chapter 11: Logarithm

- Definition of a logarithm in base b, simple logarithmic identities (change-of-base formula, addition/subtraction of logarithms)
- Natural logarithms, the number e
- Applications: Binary search, merge sort example

Sample Problem:
(AMC12-2018-A14) The solution to the equation $\log_{3x} 4 = \log_{2x} 8$, where x is a positive real number other than $\frac{1}{3}$ or $\frac{1}{2}$, can be written as $\frac{p}{q}$, where p and q are relatively prime positive integers. What is $p + q$?

(A) 5 **(B)** 13 **(C)** 17 **(D)** 31 **(E)** 35

Chapter 12: Complex Numbers

- More rigorous introduction to complex numbers
- Review of the Fundamental Theorem of Algebra, conjugate root theorem
- Polar form, Euler's formula, de Moivre's formula
- Roots of unity

Sample Problem:
(BMT-2016-Individual-5) Positive integers x, y, z satisfy $(x + yi)^2 - 46i = z$. What is $x + y + z$?