MC25G
AMC 8/MathCounts Advanced Geometry

Chapter 1: Angles

- Definitions of acute, right, obtuse, complementary, and supplementary angles
- Parallel, perpendicular, and transversal lines
- Sum of the degree measures in a triangle, different types of triangles
- Inscribed angles and arcs in circles

Sample Problem:
(Sean Shi) In triangle ABC, the measure of angle A is 42. The angle bisectors of angles B and C meet at I. Find the measure of angle BIC.

Chapter 2: Special Triangles

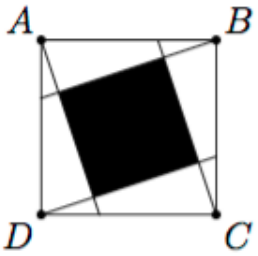
- 30-60-90 and 45-45-90 triangles
- Pythagorean theorem and Pythagorean triples

Sample Problem:
(Wanlin Li) Quadrilateral $ABCD$ has right angles at B and D. If $AC = 2\sqrt{6}, AB = \sqrt{6}$, and $AD = 2\sqrt{3}$, find the area of $ABCD$ in simplest radical form.

Chapter 3: Similarity

- Congruence and similarity axioms (SSS, SAS, ASA, AA)
- SSA is not a congruence axiom
- Angle bisector theorem

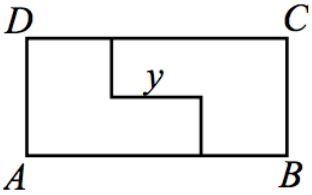
Sample Problem:
(BmMT-2012-Team-8) As pictured, lines are drawn from the vertices of a unit square to an opposite trisection point. If each triangle has legs with ratio 3:1, what is the area of the shaded region?



Chapter 4: Length-1

- Perimeter of polygons
- Triangle inequality
- Review of the Pythagorean theorem

Sample Problem:
(AMC10-2006-A7) The 8×18 rectangle $ABCD$ is cut into two congruent hexagons, as shown, in such a way that the two hexagons can be repositioned without overlap to form a square. What is y?

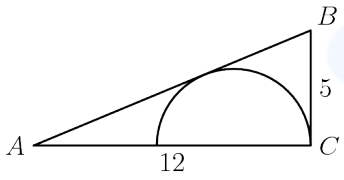


(A) 6 **(B)** 7 **(C)** 8 **(D)** 9 **(E)** 10

Chapter 5: Length-2

- Circumference of a circle
- Power of a point
- Inscribed and circumscribed circles of a triangle
- Ravi substitution

Sample Problem:
(AMC8-2017-22) In the right triangle ABC, $AC = 12$, $BC = 5$, and angle C is a right angle. A semicircle is inscribed in the triangle as shown. What is the radius of the semicircle?



(A) $\frac{7}{6}$ **(B)** $\frac{13}{5}$ **(C)** $\frac{59}{18}$ **(D)** $\frac{10}{3}$ **(E)** $\frac{60}{13}$

Chapter 6: Length-3

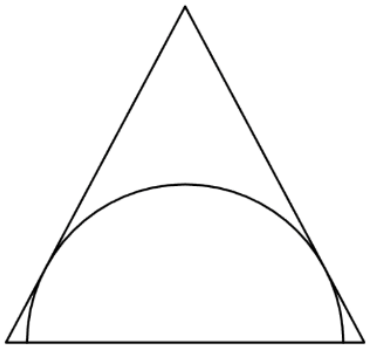
- Introduction to the mass points technique using physics concepts (levers, torque)
- Ceva's theorem and Menelaus' theorem

Sample Problem:
(Ali Gurel) In $\triangle ABC$, points D, E, and F are on BC, CA, and AB, respectively. Suppose the segments AD, BE, and CF intersect at P.
If $AF/AB = 1/3$ and $AE/AC = 1/4$, what is AP/AD?

Chapter 7: Area-1

- Unit conversions (e.g. square feet to square yards)
- Areas of simple polygons (squares, rectangles, triangles, trapezoids)
- Other formulas for the area of a triangle, including Heron's

Sample Problem:
(AMC8-2016-25) A semicircle is inscribed in an isosceles triangle with base 16 and height 15 so that the diameter of the semicircle is contained in the base of the triangle as shown. What is the radius of the semicircle?

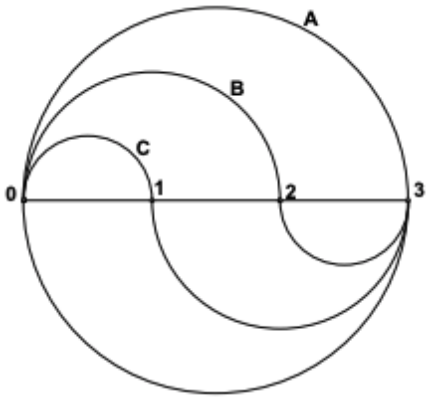


(A) $4\sqrt{3}$ **(B)** $\frac{120}{17}$ **(C)** 10 **(D)** $\frac{17\sqrt{2}}{2}$ **(E)** $\frac{17\sqrt{3}}{2}$

Chapter 8: Area-2

- Area of a circle and sector

Sample Problem:
(UNB-2008-Gr 9-24) A is a circle whose diameter is equal to 3 units. Curves B and C are respectively made from one half-circle of diameter equal to 1 unit and one half-circle of diameter equal to 2 units. What is the area of the region located between curves B and C?



(A) $\frac{3}{4}$ **(B)** $\frac{3\pi}{4}$ **(C)** 3 **(D)** 3π **(E)** None of these

Chapter 9: Analytic Geometry-1

- Cartesian coordinate system (2 dimensions)
- Slope-intercept and point-slope form of a line
- Midpoint and distance formula
- Solving geometry problems by using coordinates

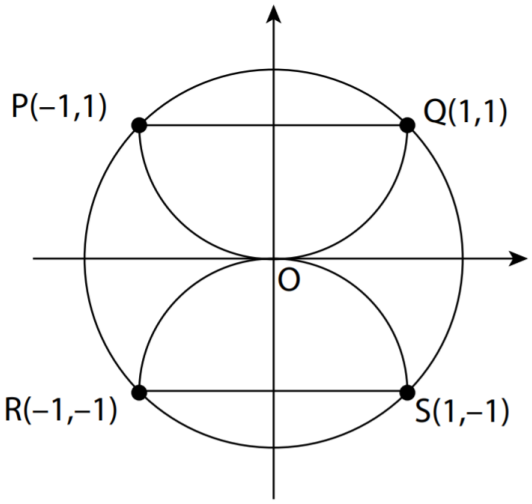
Sample Problem:
(AMC12-2018-B3) A line with slope 2 intersects a line with slope 6 at the point $(40, 30)$. What is the distance between the x-intercepts of these two lines?

(A) 5 **(B)** 10 **(C)** 20 **(D)** 25 **(E)** 50

Chapter 10: Analytic Geometry-2

- Reflecting/rotating a point in the coordinate plane
- General equation of a circle in the coordinate plane
- Area of a polygon with Shoelace formula

Sample Problem:
(AMC8-2010-23) Semicircles POQ and ROS pass through the center of circle O. What is the ratio of the combined areas of the two semicircles to the area of circle O ?



(A) $\frac{\sqrt{2}}{4}$ **(B)** $\frac{1}{2}$ **(C)** $\frac{2}{\pi}$ **(D)** $\frac{2}{3}$ **(E)** $\frac{\sqrt{2}}{2}$

Chapter 11: 3D-1

- Applications of 3D geometry in the real world
- Applying 2D geometry techniques to 3D space, 3D distance formula
- Surface area of various polyhedra, cylinders, cones, spheres

Sample Problem:
(Alexander Parr) A paper towel roll company is designing a cardboard paper towel roll. It has a circumference of 5 and a height of 48. There is a crease that starts at the bottom and spirals around the tube 4 times before reaching the top. What is the length of the spiral?

Chapter 12: 3D-2

- Volume of various 3D shapes (polyhedra, cylinders, cones, spheres)

- Volume of more complex shapes

Sample Problem:
(Alexander Parr) A cube is inscribed inside of a sphere with a radius of 1 cm. Find the volume of the region inside the sphere but outside of the cube.