MC25C
AMC 8/MathCounts Advanced Counting

Chapter 1: Addition/Multiplication Principles

- Addition (rule of sum)
- Multiplication (rule of product)

Sample Problem:
(Richard Spence) How many positive divisors of 10! are multiples of 10?

Chapter 2: Permutations

- Factorials, permutations
- Counting the number of permutations of n objects taken k at a time

Sample Problem:
(Richard Spence) The sixteen students at $C^\star$ Math Camp class split up into four groups: the Teleporters, the Transformers, the Timebenders, and the Mindbenders. Each group then decides one person to solve the number theory problems, one person to solve the algebra problems, one person to solve the geometry problems, and one person to solve the combinatorics problems, such that each student in each group solves exactly one type of problem. In how many ways can this be done?

Chapter 3: Combinations

- Difference between permutations and combinations
- How to compute combinations ("n choose k")

Sample Problem:
(A-Star) Two lines intersect at a point. 3 points are given on the first line and 4 points are given on the second line. None of the seven points are the intersection point of two lines. Find the number of triangles whose vertices are among these 7 given points.

Chapter 4: Casework

- Using casework to solve a variety of counting problems that can't be computed directly
- Use casework to break difficult problems into easier pieces

Sample Problem:
(CEMC-2012-Gauss8-24) Stones are numbered 1, 2, 3, 4, 5, 6, 7, 8, 9, 10. Three groups of stones can be selected so that the sum of each group is 11. For example, one arrangement is {1, 10}, {2, 3, 6}, {4, 7}. Including the example, how many arrangements are possible?

(A) 13 **(B)** 16 **(C)** 11 **(D)** 12 **(E)** 15

Chapter 5: Complementary Counting & Overcounting

- Applying the techniques of complementary counting or overcounting to solve problems that would be difficult otherwise

Sample Problem:
(Richard Spence) There are ten students in a class. How many ways can the teacher pair up the students into five pairs of two students each? The order of the students in each pair and the ordering of the five pairs does not matter.

Chapter 6: Counting Sets

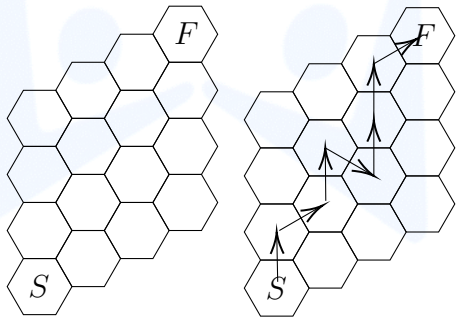
- Definitions of set, subset, size, union, and intersection
- Principle of Inclusion-Exclusion

Sample Problem:
(Sean Shi) There are 150 people at a camp. 70 of them play chess, 50 play cards, 50 play checkers, 20 play both chess and checkers, 14 play both chess and cards, and 16 play both checkers and cards. In addition, 4 play all three games. How many people play none of the games?

Chapter 7: Counting Shapes & Paths

- Counting the number of paths in a lattice grid using combinations and permutations
- Counting shapes or paths systematically (e.g. without counting manually)

Sample Problem:
(Richard Spence) A frog starts at S on the hexagonal grid of 16 tiles shown below. On any move, it can hop from its current hexagon to the hexagon above it, or either hexagon(s) on its right. An example sequence of moves is shown to the right. In how many different ways can the frog reach F using any such sequence of moves?



Chapter 8: Counting with Digits

- Various counting problems involving digits
- Palindromic numbers

Sample Problem:
(Kevin Chang) How many 3-digit numbers are there such that the hundreds digit is equal to the average of the tens and units digits?

Chapter 9: Stars and Bars

- Applying the stars and bars (or "balls and boxes") technique to solve various counting problems

Sample Problem:
(Sean Shi) How many integer solutions (w, x, y, z) are there to $w + x + y + z = 12$ such that $0 \leq w, x, y, z$; $x \leq 1$; and y is even?

Chapter 10: Binomial & Pascal's Triangle

- Binomial theorem (expanding $(x + y)^n$)
- Pascal's triangle

Sample Problem:
(Classic) Give a closed formula for the sum

$$1\binom{n}{1} + 2\binom{n}{2} + 3\binom{n}{3} + \cdots + n\binom{n}{n}.$$

Chapter 11: Probability-1

- Definition, of probability
- Sample space, independent/dependent events, disjoint events

Sample Problem:
(BmMT-2016-Individual-16) Alice rolls one pair of 6-sided dice, and Bob rolls another pair of 6-sided dice. What is the probability that at least one of Alice's dice shows the same number as at least one of Bob's dice?

Chapter 12: Probability-2

- Expected value and linearity of expectation
- Conditional probability, Bayes' theorem
- Geometric probability

Sample Problem:
(Richard Spence) In a certain country, exactly 1% of the population has a disease called algebritis. A certain drug test for algebritis claims a 99% accuracy; i.e. it returns a correct positive (or negative) result with probability 0.99.
A random participant from this country is selected. Given that his algebritis test was positive, what is the probability that he has algebritis?