

MC25A

AMC 8/MathCounts Advanced Algebra

Chapter 1: Integers & Arithmetic

- Order of operations with Integers (PEMDAS)
- Introducing Variables
- Word problems using arithmetic with integers

Sample Problem:

(AMC10-2004-A6) Bertha has 6 daughters and no sons. Some of her daughters have 6 daughters, and the rest have none. Bertha has a total of 30 daughters and granddaughters, and no great-granddaughters. How many of Bertha's daughters and granddaughters have no daughters?

- (A) 22 (B) 23 (C) 24 (D) 25 (E) 26

Chapter 2: Fractions & Decimals

- Different types of fractions (proper/improper fractions, mixed numbers, simplest form)
- Decimals with repeating/terminating digits
- Converting between fractions and decimals
- Adding, subtracting, multiplying, dividing fractions/decimals

- Telescoping sums and products
- Word problems with fractions and decimals

Sample Problem:

(AMC8-2010-21) Hui is an avid reader. She bought a copy of the best seller *Math is Beautiful*. On the first day, she read $\frac{1}{5}$ of the pages plus 12 more, and on the second day she read $\frac{1}{4}$ of the remaining pages plus 15 more. On the third day she read $\frac{1}{3}$ of the remaining pages plus 18 more. She then realizes she has 62 pages left, which she finishes the next day. How many pages are in this book?

- (A) 120 (B) 180 (C) 240 (D) 300 (E) 360

Chapter 3: Percent

- Conversions between percent and fractions/decimals
- Word problems involving percent (tax, tip, interest, etc.)
- Compound Interest
- Word problems with percent

Sample Problem:

(AMC10-2008-A8) Heather compares the price of a new computer at two different stores. Store A offers 15% off the sticker price followed by a \$90 rebate, and store B offers 25% off the same sticker price with no rebate. Heather saves \$15 by buying the computer at store A instead of store B. What is the sticker price of the computer, in dollars?

- (A) 750 (B) 900 (C) 1000 (D) 1050 (E) 1500

Chapter 4: Exponents

- Basic properties of exponents (multiplying, dividing, raising an exponent to another exponent)
- Negative exponents
- Word problems with exponents

Sample Problem:

(AMC8-2010-24) What is the correct ordering of the three numbers, 10^8 , 5^{12} , and 2^{24} ?

- (A) $2^{24} < 10^8 < 5^{12}$ (B) $2^{24} < 5^{12} < 10^8$ (C) $5^{12} < 2^{24} < 10^8$ (D) $10^8 < 5^{12} < 2^{24}$ (E) $10^8 < 2^{24} < 5^{12}$

Chapter 5: Radicals

- Square roots, cube roots, simplest radical form
- Negative/fractional exponents
- Rationalizing the denominator, simplifying radicals

Sample Problem:

(Jennifer Zhang) Simplify $\frac{\sqrt{63}}{\sqrt{315} + \sqrt{140}}$.

Chapter 6: Word Problems

- Converting a word problem into mathematical equations
- Solving two-unknown linear equations

Sample Problem:

(UNB-2008-Gr 9-7) Marina has a bank containing only pennies and nickels. If the pennies were nickels and the nickels were pennies, she would have exactly \$1.00 more. If the total value of the money in her bank is \$1.75, how many pennies does Marina have?

- (A) 25 (B) 30 (C) 40 (D) 50 (E) Not enough information

Chapter 7: Time, Travel, Work

- Unit conversions
- Distance = Rate \times Time

- Average speed, relative speed
- Problems involving the amount of work/output done

Sample Problem:

(AMC10-2008-A6) A triathlete competes in a triathlon in which the swimming, biking, and running segments are all of the same length. The triathlete swims at a rate of 3 kilometers per hour, bikes at a rate of 20 kilometers per hour, and runs at a rate of 10 kilometers per hour. Which of the following is closest to the triathlete's average speed, in kilometers per hour, for the entire race?

- (A) 3 (B) 4 (C) 5 (D) 6 (E) 7

Chapter 8: Sequences-1

- Mean, median, mode, range
- Weighted average

Sample Problem:

(Kevin Chang) Given a set of four positive integers with median 20 and mean 23, what is the smallest possible range of the set?

Chapter 9: Sequences-2

- Arithmetic and geometric sequences
- Geometric series (finite and infinite)
- Recursively defined sequences (e.g. the Fibonacci sequence)

Sample Problem:

(Kevin Chang) If the angles of an octagon are all integers and form an arithmetic sequence, find the sum of all possible values of the smallest angle. Express your answer in degrees.

Chapter 10: Functions & Operations

- Definitions of function, domain, range
- Linear functions ($f(x) = ax + b$)
- Piecewise-defined functions
- Absolute value, floor/ceiling value
- Operators

Sample Problem:

(Kevin Chang) A tank is to be filled with water. Adding 130 gallons to an empty tank fills 52% of the tank. How many gallons does the tank contain when it is completely full?

Chapter 11: Polynomials-1

- Polynomials of a single variable
- Definitions of degree, coefficient, root
- Quadratic polynomials and the quadratic formula

Sample Problem:

(Kevin Chang) Define $a \otimes b = ab - a - b$ for real numbers a, b . Then evaluate

$$(((100 \otimes 99) \otimes 98) \otimes \dots) \otimes 1.$$

Chapter 12: Polynomials-2

- Sum and product of the roots of a quadratic
- Vieta's formulas for cubic and higher degree polynomials

Sample Problem:

(Ali Gurel) Let m and n be roots of the polynomial $x^2 - 60x + 864 = 0$. Find a polynomial with roots $m + 1$ and $n + 1$.