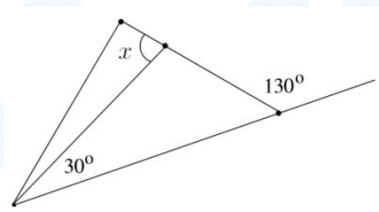
# MC20G
# AMC 8/MathCounts Basic Geometry

## Chapter 1: Angles

- Definitions of acute, right, obtuse, complementary, and supplementary angles
- Parallel, perpendicular, and transversal lines
- Sum of the degree measures in a triangle, different types of triangles
- Inscribed angles and arcs in circles

**Sample Problem:** (UNB-2018-Gr 9-3) Find the measure of the angle labeled $x$ in the diagram.



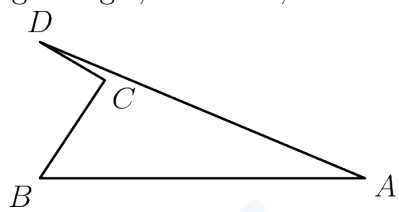

**(A)** 70° **(B)** 75° **(C)** 80° **(D)** 100° **(E)** 160°

## Chapter 2: Special Triangles

- 30-60-90 and 45-45-90 triangles

- Pythagorean theorem and Pythagorean triples

**Sample Problem:** (AMC8-2017-18) In the non-convex quadrilateral $ABCD$ shown below, $\angle BCD$ is a right angle, $AB = 12$, $BC = 4$, $CD = 3$, and $AD = 13$.



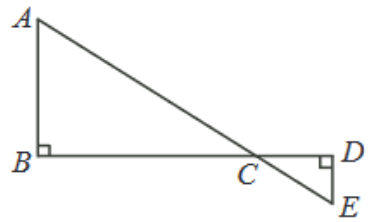

What is the area of quadrilateral $ABCD$?

**(A)** 12 **(B)** 24 **(C)** 26 **(D)** 30 **(E)** 36

# Chapter 3: Similarity

- Congruence and similarity axioms (SSS, SAS, ASA, AA)
- SSA is not a congruence axiom
- Angle bisector theorem

**Sample Problem:** (CEMC-2011-Gauss8-15) In the diagram, $AE$ and $BD$ are straight lines that intersect at $C$. If $BD = 16$, $AB = 9$, $CE = 5$, and $DE = 3$, then the length of $AC$ is




**(A)** 11 **(B)** 12 **(C)** 15 **(D)** 17 **(E)** 16

# Chapter 4: Length-1

- Perimeter of polygons

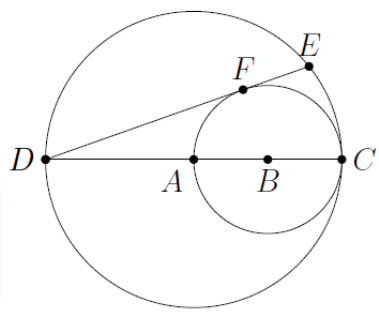
- Triangle inequality
- Review of the Pythagorean theorem

**Sample Problem:**
(A-Star) A right triangle has legs 8 cm and 15 cm. Find the shortest altitude of this triangle, in centimeters. Express your answer as a common fraction in reduced form.

# Chapter 5: Length-2

- Circumference of a circle
- Power of a point
- Inscribed and circumscribed circles of a triangle
- Ravi substitution

**Sample Problem:**
(PPP Vol8 p36 q10) In the diagram, $DC$ is a diameter of the larger circle centered at $A$, and $AC$ is a diameter of the smaller circle centered at $B$. If $DE$ is tangent to the smaller circle at $F$, and $DC = 12$, determine the length of $DE$.




# Chapter 6: Length-3

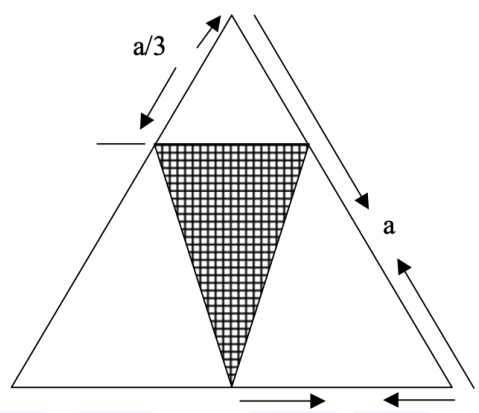
- Introduction to the mass points technique using physics concepts (levers, torque)
- Ceva's theorem and Menelaus' theorem

**Sample Problem:**
(Ali Gurel) $P$ is a point inside triangle $\triangle ABC$. The lines $AP$, $BP$, and $CP$ intersect the sides of the triangle at $D$, $E$, and $F$, respectively. If $AP = PD$ and $BD = 2DC$, what is $BF/FA$?

# Chapter 7: Area-1

- Unit conversions (e.g. square feet to square yards)
- Areas of simple polygons (squares, rectangles, triangles, trapezoids)
- Other formulas for the area of a triangle, including Heron's

**Sample Problem:**
(UNB-2000-Gr 9-22) In the figure below, the area of the shaded triangle is $2\sqrt{3}$. If the large triangle and the small upper triangle are equilateral, what is the value of $a$?




**(A)** 2 **(B)** 2.5 **(C)** 3 **(D)** 6 **(E)** None of these

# Chapter 8: Area-2

- Area of a circle and sector

**Sample Problem:**
(Classic) Three circles with radius 1 meter are pairwise tangent to each other. Find the area that is enclosed in between the three circles.

# Chapter 9: Analytic Geometry-1

- Cartesian coordinate system (2 dimensions)
- Slope-intercept and point-slope form of a line
- Midpoint and distance formula
- Solving geometry problems by using coordinates

**Sample Problem:**
(BmMT-2012-Ciphering-26) The lines $y = 3x$ and $x = 4$ form a right triangle with the $x$-axis. Find the slope of a line through the origin that bisects the triangle into two portions of equal area.

# Chapter 10: Analytic Geometry-2

- Reflecting/rotating a point in the coordinate plane
- General equation of a circle in the coordinate plane
- Area of a polygon with Shoelace formula

**Sample Problem:**
(PPP Vol5 p26 q11) Find the coordinates of all points in the Cartesian plane that are equidistant from the $x$-axis, $y$-axis, and the point $(2, 1)$.

# Chapter 11: 3D-1

- Applications of 3D geometry in the real world
- Applying 2D geometry techniques to 3D space, 3D distance formula
- Surface area of various polyhedra, cylinders, cones, spheres

**Sample Problem:**
(BmMT-2012-Ciphering-20) What is the surface area of a cube inscribed in a sphere with surface area $8\pi$?

# Chapter 12: 3D-2

- Volume of various 3D shapes (polyhedra, cylinders, cones, spheres)
- Volume of more complex shapes

**Sample Problem:**
(BmMT-2012-Ciphering-24) An $8 \times 11$ sheet of paper is rolled up so that the 11-inch edges align. Find the volume of the resulting cylinder.