

MC20C

AMC 8/MathCounts Basic Counting

Chapter 1: Addition/Multiplication Principles

- Addition (rule of sum)
- Multiplication (rule of product)

Sample Problem:

(UNB-2008-Gr 9-16) How many ways can the numbers 1, 2, 3, 4 and 5 be placed in a line so that neither 1 nor 5 occupy either the first or the last place in the sequence?

- (A) 6 (B) 24 (C) 36 (D) 54 (E) 72

Chapter 2: Permutations

- Factorials, permutations
- Counting the number of permutations of n objects taken k at a time

Sample Problem:

(CEMC-2006-Gauss8-19) Bethany, Chun, Dominic, and Emily go to the movies. They choose a row with four consecutive empty seats. If Dominic and Emily must sit beside each other, in how many different ways can the four friends sit?

- (A) 6 (B) 5 (C) 12 (D) 30 (E) 3

Chapter 3: Combinations

- Difference between permutations and combinations
- How to compute combinations (“n choose k”)

Sample Problem:

(Richard Spence) A math club is trying to select a subset of its members to form a committee. It notices that the number of possible 4-member committees equals the number of possible 6-member committees. How many members are in the math club?

Chapter 4: Casework

- Using casework to solve a variety of counting problems that can’t be computed directly
- Use casework to break difficult problems into easier pieces

Sample Problem:

(Jafar Jafarov) A committee of five people is selected from seven men and six women. How many ways are there to select the committee so that there are at least two men and two women on the committee?

Chapter 5: Complementary Counting & Overcounting

- Applying the techniques of complementary counting or overcounting to solve problems that would be difficult otherwise

Sample Problem:

(AMC8-2016-17) An ATM password at Fred’s Bank is composed of four digits from 0 to 9, with repeated digits allowable. If no password may begin with the sequence 9, 1, 1, then how many passwords are possible?

- (A) 30 (B) 7290 (C) 9000 (D) 9990 (E) 9999

Chapter 6: Counting Sets

- Definitions of set, subset, size, union, and intersection
- Principle of Inclusion-Exclusion

Sample Problem:

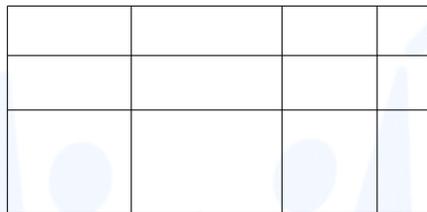
(Jafar Jafarov) There are 25 students in a class. 12 of them are on a football team and 14 are on a soccer team. If 3 students are on neither of these teams, how many students are on both football and soccer teams?

Chapter 7: Counting Shapes & Paths

- Counting the number of paths in a lattice grid using combinations and permutations
- Counting shapes or paths systematically (e.g. without counting manually)

Sample Problem:

(Richard Spence) How many rectangles of any size are in the figure below?



Chapter 8: Counting with Digits

- Various counting problems involving digits
- Palindromic numbers

Sample Problem:

(BmMT-2016-Team-8) A seven digit number is called “bad” if exactly four of its digits are 0 and the rest are odd. How many seven digit numbers are bad?

Chapter 9: Stars and Bars

- Applying the stars and bars (or “balls and boxes”) technique to solve various counting problems

Sample Problem:

(Victor Hakim) How many positive integer solutions (x, y, z, w) are there to $x + y + z + w = 15$?

Chapter 10: Binomial & Pascal’s Triangle

- Binomial theorem (expanding $(x + y)^n$)
- Pascal’s triangle

Sample Problem:

(Richard Spence) Simplify

$$\binom{100}{0} + 2\binom{100}{1} + 4\binom{100}{2} + 8\binom{100}{3} + \dots + 2^{100}\binom{100}{100}.$$

Chapter 11: Probability-1

- Definition, of probability
- Sample space, independent/dependent events, disjoint events

Sample Problem:

(Richard Spence) Four fair six-sided dice are rolled. What is the probability that the largest number rolled is at least 4? Express your answer as a common fraction in reduced form.

Chapter 12: Probability-2

- Expected value and linearity of expectation
- Conditional probability, Bayes’ theorem
- Geometric probability

Sample Problem:

(Victor Hakim) x and y are two positive real numbers chosen randomly and uniformly in the interval $[0, 2]$. What is the probability that $x^2 + y^2 \geq 1$ and $y \geq x$?

