

MC20A

AMC 8/MathCounts Basic Algebra

Chapter 1: Integers & Arithmetic

- Order of operations with Integers (PEMDAS)
- Introducing Variables
- Word problems using arithmetic with integers

Sample Problem:

(AMC10-2002-A6) Cindy was asked by her teacher to subtract 3 from a certain number and then divide the result by 9. Instead, she subtracted 9 and then divided the result by 3, giving an answer of 43. What would her answer have been had she worked the problem correctly?

- (A) 15 (B) 34 (C) 43 (D) 51 (E) 138

Chapter 2: Fractions & Decimals

- Different types of fractions (proper/improper fractions, mixed numbers, simplest form)
- Decimals with repeating/terminating digits
- Converting between fractions and decimals
- Adding, subtracting, multiplying, dividing fractions/decimals

- Telescoping sums and products
- Word problems with fractions and decimals

Sample Problem:

(UNB-2016-Gr 9-12) A collection of coins was shared. Mary received $\frac{1}{3}$ of the coins, Amir received $\frac{1}{5}$ of the coins, and Samita received $\frac{1}{6}$ of the coins. The remaining 36 coins were given to Troy. How many coins were in the entire collection?

- (A) 84 (B) 90 (C) 108 (D) 120 (E) 144

Chapter 3: Percent

- Conversions between percent and fractions/decimals
- Word problems involving percent (tax, tip, interest, etc.)
- Compound Interest
- Word problems with percent

Sample Problem:

(CEMC-2000-Gauss7-13) Karl had his salary reduced by 10%. He was later promoted and his salary was increased by 10%. If his original salary was \$20,000, what is his present salary?

- (A) \$16,200 (B) \$19,800 (C) \$20,000 (D) \$20,500 (E) \$24,000

Chapter 4: Exponents

- Basic properties of exponents (multiplying, dividing, raising an exponent to another exponent)
- Negative exponents
- Word problems with exponents

Sample Problem:

(AMC10-2002-A3) According to the standard convention for exponentiation,

$$2^{2^{2^2}} = 2^{(2^{(2^2)})} = 2^{16} = 65,536$$

If the order in which the exponentiations are performed is changed, how many other values are possible?

- (A) 0 (B) 1 (C) 2 (D) 3 (E) 4

Chapter 5: Radicals

- Square roots, cube roots, simplest radical form
- Negative/fractional exponents
- Rationalizing the denominator, simplifying radicals

Sample Problem:

(Richard Spence) Compute the value of $(\sqrt{12} + \sqrt{75} + \sqrt{108})^2$.

Chapter 6: Word Problems

- Converting a word problem into mathematical equations
- Solving two-unknown linear equations

Sample Problem:

(UNB-2010-Gr 9-21) Farmer Fred said to Farmer John: “If you sell me 45 hectares of land, I will have twice as much land as you.” Then Farmer John said to Farmer Fred: “If you sell me 45 hectares of land, I will have just as much land as you.” How many hectares of land does farmer Fred have?

- (A) 135 (B) 180 (C) 225 (D) 270 (E) 315

Chapter 7: Time, Travel, Work

- Unit conversions
- Distance = Rate \times Time
- Average speed, relative speed
- Problems involving the amount of work/output done

Sample Problem:

(Richard Spence) Richard goes on a 6-mile jog one morning. He jogs the first two miles at an average speed of 6 mph. He progressively slows down; his average speed during the next two miles is 4 mph. He walks the remaining two miles at an average speed of 3 mph. What is Richard's average speed, in miles per hour?

Chapter 8: Sequences-1

- Mean, median, mode, range
- Weighted average

Sample Problem:

(CEMC-2016-Gauss8-16) The mean (average) of a set of six numbers is 10. If the number 25 is removed from the set, the mean of the remaining numbers is

- (A) 6 (B) 7 (C) 8 (D) 9 (E) 10

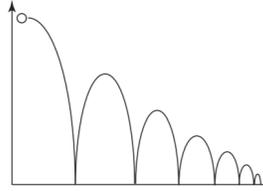
Chapter 9: Sequences-2

- Arithmetic and geometric sequences
- Geometric series (finite and infinite)
- Recursively defined sequences (e.g. the Fibonacci sequence)

Sample Problem:

(AMC8-2008-12) A ball is dropped from a height of 3 meters. On its first bounce it rises to a height of 2 meters. It keeps falling and bouncing to $\frac{2}{3}$ of the height it reached in the previous bounce. On which bounce will it not rise to a height of 0.5

meters?



(A) 3 (B) 4 (C) 5 (D) 6 (E) 7

Chapter 10: Functions & Operations

- Definitions of function, domain, range
- Linear functions ($f(x) = ax + b$)
- Piecewise-defined functions
- Absolute value, floor/ceiling value
- Operators

Sample Problem:

(Sean Shi) Let $a@b = (a - 1)(b - 1) + 1$. Find $((3@4)@5)@6$.

Chapter 11: Polynomials-1

- Polynomials of a single variable
- Definitions of degree, coefficient, root
- Quadratic polynomials and the quadratic formula

Sample Problem:

(Ali Gurel) Let m and n be roots of the polynomial $x^2 - 28x + 192$. Find a polynomial with roots $-m$ and $-n$.

Chapter 12: Polynomials-2

- Sum and product of the roots of a quadratic
- Vieta's formulas for cubic and higher degree polynomials

Sample Problem:

(Ali Gurel) Let a and g be roots of the polynomial $x^2 - 60x + 899 = 0$. What is $a^2 + g^2$?

