MC15C
Pre-MathCounts Counting

Chapter 1:   Addition Principle

- Counting the number of elements in a set or sequence quickly (e.g. how many multiples of 5 are between 100 and 1000 inclusive?)

- Using addition to count the number of ways to accomplish a task

Sample Problem:
(Hope Chen) How many numbers between 1 and 32 (inclusive) are divisible by 3?

Chapter 2:   Multiplication Principle

- Using multiplication to count the number of ways to accomplish a task

Sample Problem:
(Richard Spence) In poker, a five-card hand is called a *four-of-a-kind* if there are four cards of the same rank, and a fifth card of different rank. An example four-of-a-kind is A♣A♠A♢A♡4♠. How many different four-of-a-kind hands are there?

Chapter 3:   Permutations

- Definition of the factorial (n!)

- Permutations - Finding the number of ways to choose k items from a set of n items where the ordering of items is important

Sample Problem:
(Rohan Cherukuri) Eight students have to get into line. How many ways can they do this, if the oldest one must be at the front?

Chapter 4:   Combinations

- How to compute n choose k

- Difference between combinations and permutations taken k at a time

- Number of ways to form a committee using combinations

Sample Problem:
(Jocelyn Zhu) Victor has 6 friends. He has a movie pass that allows him to bring 3 more friends for free. How many different groups of three friends can Victor choose to go to the movies?

Chapter 5:   Casework

- Using casework to solve a variety of counting problems

Sample Problem:
(CEMC-2006-Gauss7-24) A triangle can be formed having side lengths 4, 5 and 8. It is impossible, however, to construct a triangle with side lengths 4, 5 and 10. Using the side lengths 2, 3, 5, 7 and 11, how many different triangles *with exactly two equal sides* can be formed?

(A) 8   **(B)** 5   **(C)** 20   **(D)** 10   **(E)** 14

Chapter 6:   Complementary Counting

- Using complementary counting - we count the number of ways we don't want first, then subtract this result from the total

Sample Problem:
(Rohan Cherukuri) A school needs to select a debate team of size 3 from its pool of 10 people. Abraham and Stephen are both in this pool. In how many ways can at least one of them be selected for the 3 person team?

Chapter 7:   Overcounting

- Using the technique of overcounting by counting more than what we need, then subtracting or dividing to account for the extra cases considered

Sample Problem:
(Jocelyn Zhu) How many different ways can you arrange the letters in the word "ALPHA"?

Chapter 8:   Counting Sets

- Simple definition of a set in mathematics (finite sets, including the empty set)

- Union and intersection of two sets

- Venn diagrams, Principle of Inclusion-Exclusion, other counting problems involving sets

Sample Problem:
(MathCounts-2014-Chapter-Countdown-54) Among the 65 cheerleaders at an Austin middle school, 25 were Cowboys fans, 42 were Texans fans, and 6 were not fans of either team. How many cheerleaders were fans of both teams?

Chapter 9:   Counting Shapes & Paths

- Counting the number of shapes or paths systematically (e.g. using combinations)

Sample Problem:
(Abby Berry) A class wants to walk to Walmart to buy some Jolly Ranchers. Imagine that the path they are walking is along a grid. They are starting at (0,0) and Walmart is at (3,4), and they can only walk straight or go right. Also, they may only turn at lattice points. How many ways are there for them to get to Walmart?

Chapter 10:   Counting with Digits

- Counting problems involving digits of a number

- Palindromic numbers

Sample Problem:
(Rohan Cherukuri) How many 6 digit palindromes have a 0 in them?

Chapter 11:   Probability-1

- Definition of probability (number of desired outcomes divided by the total number of outcomes)

- Solving probability problems using other counting techniques (e.g. complementary counting)

Sample Problem:
(MathCounts-2012-School-Sprint-11) If you toss two standard six-sided dice, what is the probability that you will get a 3 on at least one die? Express your answer as a common fraction.

Chapter 12:   Probability-2

- Independent versus dependent events

Sample Problem:
(Richard Spence) A standard 52-card deck is shuffled, and the top card is drawn. This card is shuffled back into the deck, and a second card is drawn. What is the probability that the first card is an Ace and the second card is a spade?